THE CALCULATION OF THE NORTH BANTEN GEOID FROM GRAVITY RESOLUTION OF 5 BY 5 KM

Dadan Ramdani[1,2], Eki Riyanti Suyatno[1], Yadi Aryadi[1]

[1]Badan Informasi Geospasial, Jl Raya Jakarta Bogor km 46 Cibinong, Indonesia
email: dadan.ramdani@big.go.id

[2]Universitas Pakuan, Jl. Pakuan PO Box 452 Bogor 16143 Jawa Barat Indonesia

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ABSTRACT: The use of GNSS altitude requires geoid height information so that the elevation information can be converted into orthometric altitude. This orthometric height is usually used for practical purposes. In 2015 the Jakarta geoid was created and has an accuracy of 0.080 m. Airborne gravimetry carried out for the whole of Java Island did not cover the DKI Province because there were obstacles from Airnav. with terrestrial gravimetry carried out in this region outside the area measured in 2012. In this paper the Global Geopotential Model used is gif48. The use of the "delete and restore" method as well as the Stokes and FFT kernels to speed up calculations is carried out in calculating the geoid in the Jakarta area. The geoid produced was verified with 11 points in DKI Jakarta Province. This verification yields a standard deviation of 0.166 m and a mean square root of 0.411 m.

Introduction

Indonesia uses an orthometric height system in accordance with the Head of BIG Regulation no 15 of 2013 concerning the 2013 Indonesian Geospatial Reference System (SRGI2013). Where the height refers to the geoid height. The geoid height must be from the measurement of terrestrial gravity which is tied to the gravity control net (JKG) that must be bound to IGSN71 (BIG, 2013) .

Geoid is needed in large scale map making, where this large scale creation uses high data from LIDAR. Where the height of the LIDAR is corrected by using a Geoid so that it becomes an orthometric height. In the use of the height for GNSS it is necessary to transform its height into orthometric height. To make this height an orthometric height a geoid height is required. Thus, GNSS users really need Geoid to get their orthometric height.

Orthometric height (H) can be obtained from the ellipsoid height (h) corrected by geoid (N) (Rummel, 1992) . From equation 1, it can be seen that the orthometric height is the difference between the ellipsoid height and the geoid undulation (Figure 1).

$$H = h - N \quad (1)$$

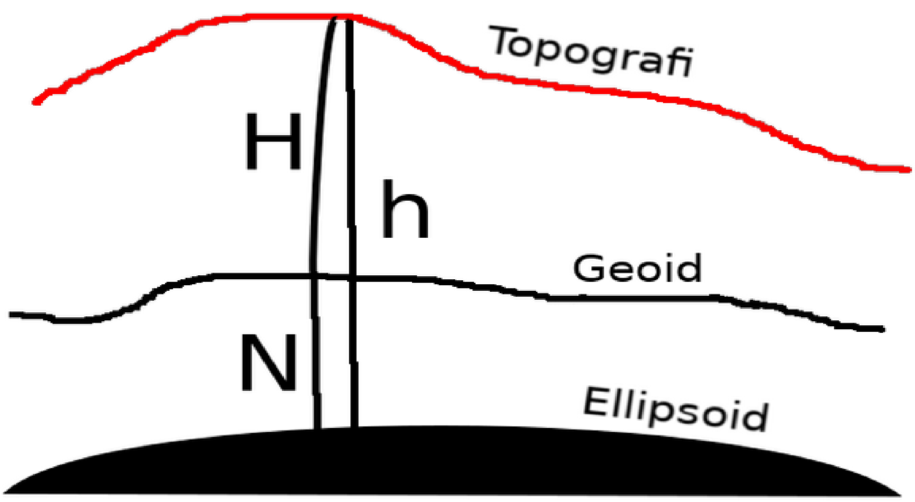

Figure 1: Relationship between Orthometric, Ellipoid, and Geoid Heigth

There are several methods for obtaining the geoid undulation namely the geometric method and the gravimetric method. In the geoid geometric method it is calculated from a combination of satellite position elevation data (GNSS) with leveling measurements, while in the gravimetric method, the geoid is calculated from terrestrial gravity data and global geopotential models (gravity potential coefficient).

Ries et al. Made the GGM in 2011 from the GRACE and GOCE satellite data called GIF48 (Ries et al., 2011). For the Jakarta area, based on the GIF48 data research, it produces the most optimal geoid (Dadan Ramdani et al., 2014). so that the GGM data from GIF48 is used to calculate the long wave geoid.

From the gravity data measured in 2010, it was obtained a geoid with an accuracy of 0.080 m (D. Ramdani et al., 2015). This accuracy is obtained by comparing the geoid from the calculated gravity data with the geoid from the reduction of the orthometric height data from the flat slice measurement with the ellipsoid height data from the GNSS measurement.

In 2019 the gravity observations for the DKI Jakarta area were expanded from the Banten area to the Karawang area carried out by the Geodesy and Geodynamic Control Network Center. With a measurement resolution of 5.0 km for each point. This resolution is different from that in the DKI Jakarta area (D. Ramdani et al., 2015), which is 1.5 km, can be seen in Figure 2. In this study a geoid will be made based on these data.

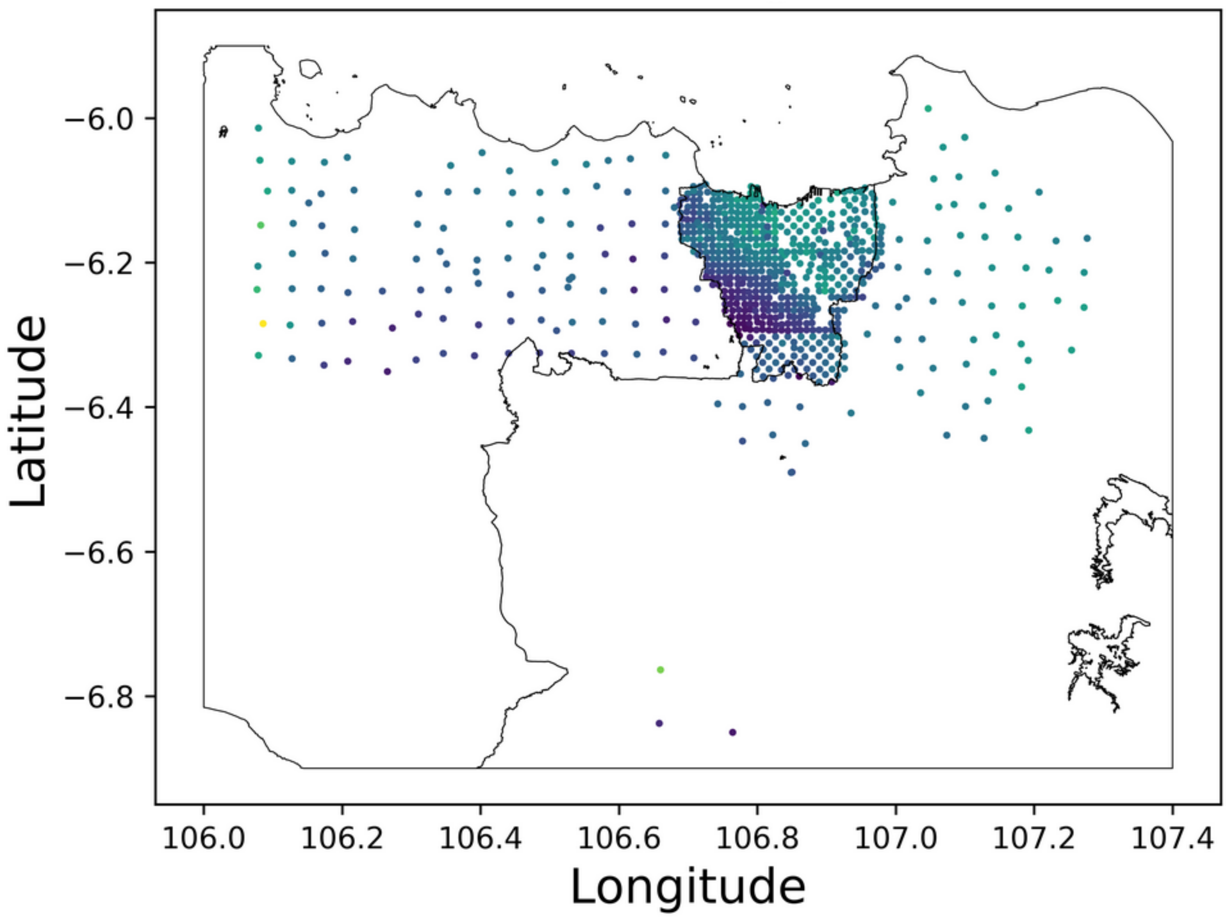

Figure 2: Distribution of Gravity Points in 2010 and 2019

Method

Geoid is formed by 3 wave components, namely long, medium and short waves. Long waves were contributed by GGM whereas medium waves were contributed by local gravity and short waves by topographic effect corrections.

Medium waves were calculated from local gravity by using the Brun equation, In the Brun equation the Geoid from the local gravity observations can be formulated according to equation (2) (Hofmann-Wellenhof & Moritz, 2005) .

$$N=\frac{T-(W-U)}{\gamma} \quad (2)$$

Based on defined data on the earth's surface and on the geoid surface as the boundary plane. A mathematical model for geoid can be determined so that it can be formulated in the form of a Boundary Value Problem (BVP) with use of a Laplace differential equation of gravitational potential disturbance that is limited by conditions. BVP in geoid calculations is also often called the Geodetic Boundary Value Problem or GBVP.

The geoid is geometrically corrected against the theoretical equipotential plane, namely the ellipsoid plane. This field is known as the normal earth. And this ellipsoid field is also the reference field. In equation (2) potential disturbance (T) or also often called disturbance potential outside the geoid surface is a harmonic function so that it satisfies the Laplace equation (Hofmann-Wellenhof & Moritz, 2005) in accordance with equation (3).

$$\Delta T=\frac{\delta^2 T}{\delta x^2}+\frac{\delta^2 T}{\delta y^2}+\frac{\delta^2 T}{\delta z^2}=0 \quad \text{.........(3)}$$

The potential disturbance T will obtain a single solution requiring one boundary condition equation (Rummel, 1992). And by bringing the geoid as a spherical with constant radius R, the boundary condition is in accordance with equation (4).

$$\Delta g=-\frac{\delta T}{\delta R}-\frac{2T}{R} \quad \text{.........(4)}$$

where Δg is the gravity anomaly which is defined as the difference between the gravity in the geoid g and the normal gravity γ according to equation (5).

$$\Delta g=g-\gamma \quad \text{.........(5)}$$

Δg can be obtained from the measurement of the gravity reduced to the geoid surface and the calculation of the normal gravity at the geoid point. This is a solution to the geodetic boundary value problem with the robin problem(Prijatna, 2010). In this geoid calculation, the boundary used is the geoid so that the Stokes equation can be used (Hofmann-Wellenhof & Moritz, 2005). Thus the geoid can be written according to equation (6) which is often called the Stokes integral.

$$N(\phi,\lambda)=\frac{R}{4\pi\gamma}\iint_{\sigma} S(\psi)\Delta g(\phi',\lambda')\,d\sigma \quad \text{.........(6)}$$

S (ψ) is a Stokes function which can be represented by equation (7) or (8)

$$S(\psi)=\sum_{n=0}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi) \quad \text{.........(7)}$$

or

$$S(\psi)=\frac{1}{\sin\frac{\psi}{2}}-6\sin\frac{\psi}{2}+1-5\cos\psi-3\cos\psi\ln\left(\sin\frac{\psi}{2}+\sin^2\frac{\psi}{2}\right) \quad \text{.........(8)}$$

ψ is the angular distance on the surface of the globe.

Gravity measurement data must be reduced to geoid except in ocean areas. Thus, the information on earth mass density and orthometric height above the geoid surface is needed to reduce gravity data to the geoid.

There are several ways to calculate equation (6) from direct calculations or using the FFT or collocation method. All of these calculation methods have their advantages and disadvantages. Equation (6) is very time consuming, to increase it can use the Fast Fourier Transformation (FFT) technique (Jekeli, 1982) . FFT requires data in a regular grid form. The FFT generates data that fits the data grid entered.

This grid data is obtained from the observed gravity point data using the Tri Interpolation Linear method found in the python matplotlib plugin (Kiusalaas, 2013).

To calculate the geoid using the Stokes integral requires gravity throughout the earth. To be able to use the integral of the stoke in a limited area, it requires GGM data as a contribution of long waves.

The long waves contributed by GGM have a big contribution. The equation of GGM according to Heiskanen and Moritz (Heiskanen & Moritz, 1967) can be seen in equation (9).

$$N_{GM}(\varphi,\lambda)=\frac{GM}{r\gamma}\sum_{n=2}^{N_{max}}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right)P_{nm}(\sin\phi) \quad (9)$$

where φ and λ are the coordinates of the sphere to be calculated, GM is the gravitational constant (including the mass of the earth's atmosphere), r is the distance from the center of the earth to the point of calculation, γ is the normal gravity, Nmax is the maximum GGM number of degrees m and order n, and C and S are components of the harmonic coefficient, and P is the Legendre polynomial function.

Stokes integral in the calculation is discrete, so it can be calculated by addition using discrete data. The result of the Geoid skhir is the sum of all wavelengths according to equation (10). However, all the contributions from the results of this geoid on the gravity data must be removed first before calculating the local geoid according to equation (11). This theory is often called remove and restore, all contributions are first subtracted from the gravity measurement results and then put back in the geoid calculation.

$$N_{total}=N_{GM}+N_{TC}+N_{lokal} \quad (10)$$

$$\Delta g_{total}=\Delta g_{GM}+\Delta g_{TC}+\Delta g_{lokal} \quad (11)$$

The gravity data used in this study is the gravity data measured in 2010 in the DKI Jakarta province with a data density of about 1.5 km using the Lacoste & Romberg gravimeter types 700 and 956. And the gravity data measured in 2019 with Data density of about 5 km (Figure 2) using the Scintrex type CG5 gravimeter. From the results of these two measurements.

Results and Discussion

The global contribution from the observed data is first removed to produce the residual gravity data, and then a grid spaced 5 km north and west is created, resulting in a point with dimensions of 78 x 24 (Figure 3). with the coordinates of the lower left end of 6.36525 LS, 106.12719 BT, and the upper right corner of 6.059767 LS, 107.14359 BT, the space for latitude is 0.013200 degrees and for longitude is 0.013282 degrees.

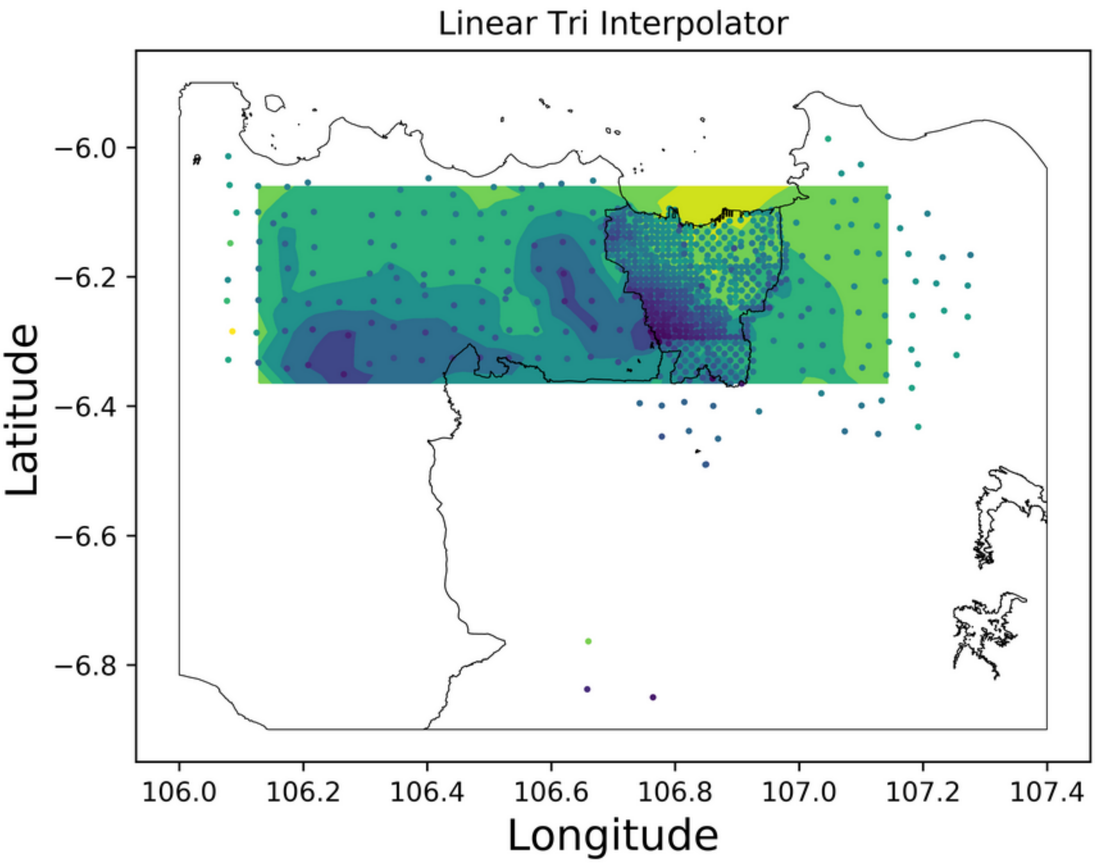

Figure 3: Contours of Tri Interpolation Linear Grid gravity data

The grid data is then used to calculate the geoid by using the FFT method from Tscherning's (Tscherning & Rapp, 1974) SPFOUR software from Gravsoft. Geoid from the results of these calculations can be seen in Figure 4.

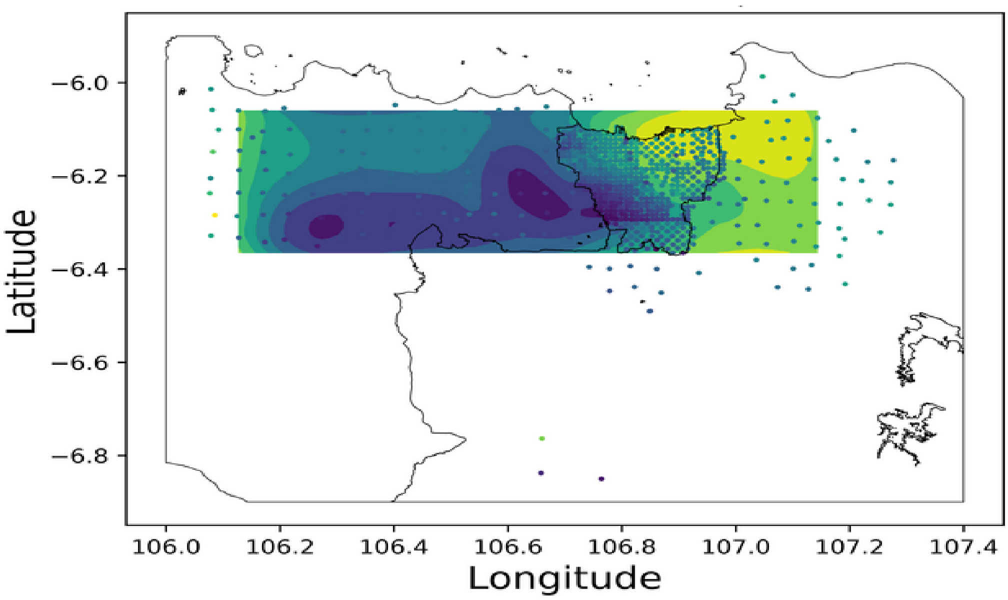

Figure 4: Contour from the spfour of gravity grid data

The results obtained were then added with the geoid from GGM GIF48 to produce geoids for the Jakarta and surrounding areas, which contours can be seen in Figure 5

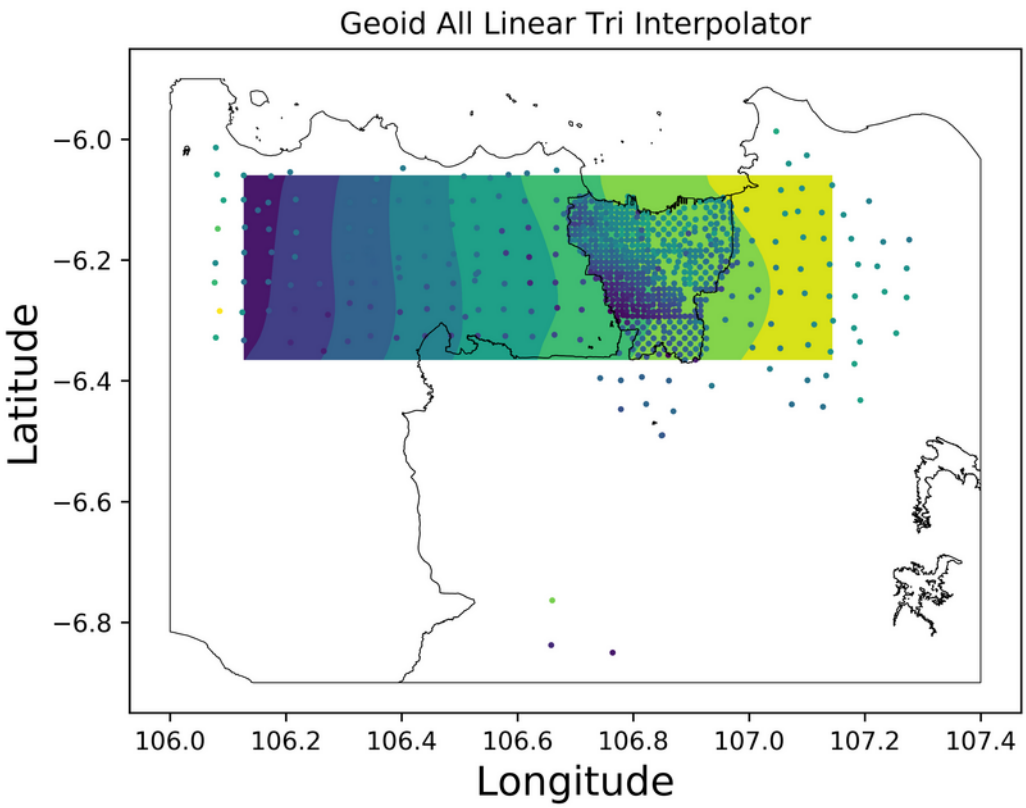

Figure 5: Geoid results in the Jakarta and surrounding areas

The quality of our local geoid model can be seen using a procedure using the 11 points in Jakarta. The 11 points have an ellipsoidal height (h) and an orthometric height (H). So that with these two heights, a geometric geoid can be calculated. The distribution of the sample is depicted in Figure 6, while the data about the validation point is shown in Table 1.

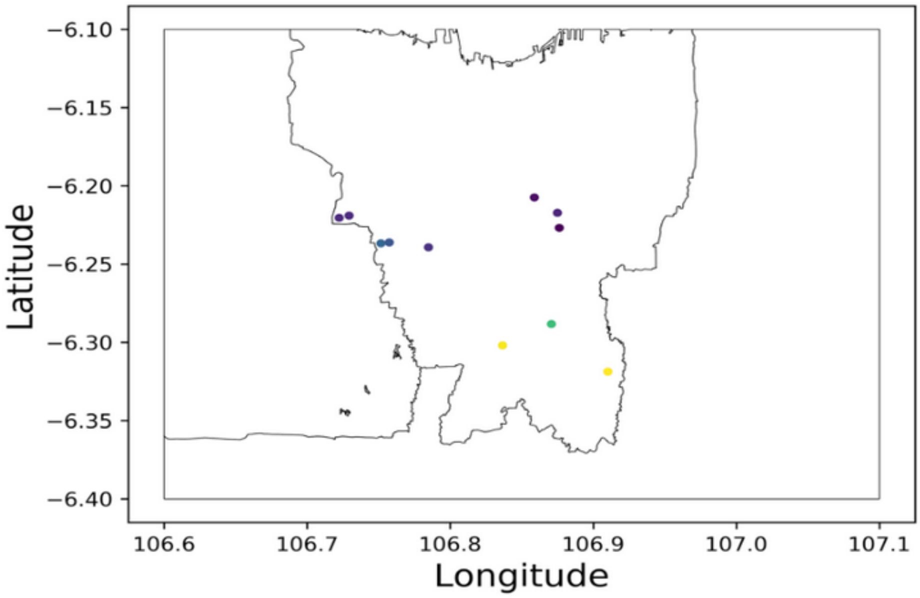

Figure 6: Distribution map of the validation points

Table 1: Validation points with position and elevations

No	Longitude	Latitude	Height (ellp)	Height (Ortho)
1	-6.2172259	106.8749299	34.576	15.499
2	-6.3186845	106.9101893	60.365	41.233
3	-6.2204172	106.7223984	34.627	15.807
4	-6.2190019	106.7293226	34.562	15.912
6	-6.2361289	106.7574098	38.923	20.050
7	-6.3019064	106.8366012	59.962	40.871
10	-6.2367149	106.7515594	40.502	21.762
11	-6.2073957	106.8588015	32.523	13.628
12	-6.2392342	106.7848056	35.204	16.427
13	-6.2882639	106.8707579	51.110	32.018
16	-6.2268139	106.8762478	31.517	12.623

This local geoid model is verified using these validation points. This verification yields a standard deviation of 0.116 m and an RMS of 0.411 (table 2). When compared with the standard deviation of the previous results by Ramdani et al. (D. Ramdani et al., 2015) for 0.080 m the verification result is still below it, this is because the resolution is different. Meanwhile, the RMS value from this study was smaller than the previous results by Ramdani et al. (D. Ramdani et al., 2015) is 0.626 m, this indicates that there is a difference in reference. The complete verification results are shown in Table 1. Although the accuracy is smaller, the data retrieval time is faster and the coverage is wider and more economical.

Table 2: Difference of Geoid Geometrc and Geoid Gravimetric

No	Geoid Geometric	Geoid Gravimetric	Diff
1	19.077	18,721	-0,355
2	19.132	18,844	-0,288
3	18.819	18,224	-0,596
4	18.650	18,245	-0,406

No	Geoid Geometric	Geoid Gravimetric	Diff
6	18.873	18,314	-0,559
7	19.091	18,606	-0,486
10	18.740	18,296	-0,444
11	18.895	18,682	-0,213
12	18.777	18,400	-0,377
13	19.092	18,684	-0,408
16	18.894	18,717	-0,177
		StDev	0,116
		RMS	0,411

Conclusion

This Jakarta geoid, whose calculations use 5 x 5 km gridded gravity measurement data, has a standard deviation of 0.116 m and an RMS of 0.411 m. This result is slightly worse than our previous local geoid model based on 1.5 x 1.5 km gravity data. This could be due to differences in the gravity measurement method from 1.5 x 1.5 km to 5 x 5 km.

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