

A Comparison of Linear Unmixing Algorithms through Remotely Sensed Images for Water Resources

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Abstract

Every feature on the earth surface emits unique radiance which could easily be interpreted by spectral signature. However, it becomes challenging while interpreting through digital imagery as its individual pixel or its spectral resolution represents the reflected radiance as a sum composite of all the features. This results into mixing of features in a pixel leading to loss of individuality. Mixing noticeably decreases with high resolution spectral data. But it still persists and depends on how the feature is arranged on the ground. The unavailability and non-accessibility of high resolution imagery of target area lead to the process of spectral unmixing and sub-pixel classification. In digital image processing, spectral unmixing is a common phenomenon which decomposes reflectance spectra into a set of given end member spectra. In this paper, various standard methodologies such as Fuzzy, Distance and Least square to Orthogonalizations were applied and compared on the basis of quantitative analysis on water features to identify the best method. The experiments were carried on a set of Landsat-8 data to evaluate the performance of spectral unmixing. The Distance based unmixing proves to be the best method for unmixing water features as it covers the entire area.

1. Introduction

Remote sensing is the process of gathering information about an object or phenomenon without making physical contact with the object in contrast to on site observation. Remote sensing is used in numerous fields, including geography and most earth science disciplines (e.g., hydrology, ecology, oceanography, glaciology, and geology). It also has military, intelligence, commercial, economic, planning, and humanitarian applications. In modern usage, the term generally refers to the use of aerial sensor technologies to detect and classify objects on earth (both on the surface, and in the atmosphere and oceans) by means of propagated signals (e.g. electromagnetic radiation). It may be split into active remote sensing (when a signal is first emitted from aircraft or satellites) or passive (e.g. sunlight) when information is merely recorded.

Spectral mixing is a common term in remote sensing and it is inevitable when images are taken of a heterogeneous surface, or from a low resolution space borne sensor and also by various factors when signal is received by the imaging spectrometer for a pixel. There are two types of pixels present in an image shown in Fig 1.

- a) Pure pixel: Pure Pixels are the pixels which represent a single class. Pure pixels represent areas covered by a single component type. The first step to identify and resolving mixed pixels is to find pure pixels of that image. Pure class pixels are the key input to the various approaches used to resolving or un-mixing problem.

b) Mixed Pixel: Mixed pixels are the pixels which are not occupied by a homogeneous class. These pixels represent more than one class. Mixed pixels are created in digital images. Mixed pixels occur at the boundary of the areas, or along long linear features, such as sea and rocky area, where contrasting brightness are immediately adjacent to one another.

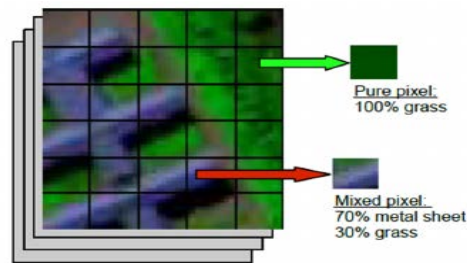


Fig 1. Pure and Mixed pixels

Mixing is more commonly present in lower spatial resolution data as compared to fine or higher spatial resolution. For example let's take a satellite image of Landsat-8 having resolution of 30m and MODIS satellite image having 250m resolution. The later one has more spectral mixing as its reflected radiance are coming from 250 m² area consisting of many features as compared to 30 m² area of the former one due to its coarse resolution. The mixing phenomenon has been clearly depicted in the Fig. 1. Sometimes mixed pixels can also results from when distinct materials are combined into a homogeneous mixture. Therefore, linear unmixing comes into play.

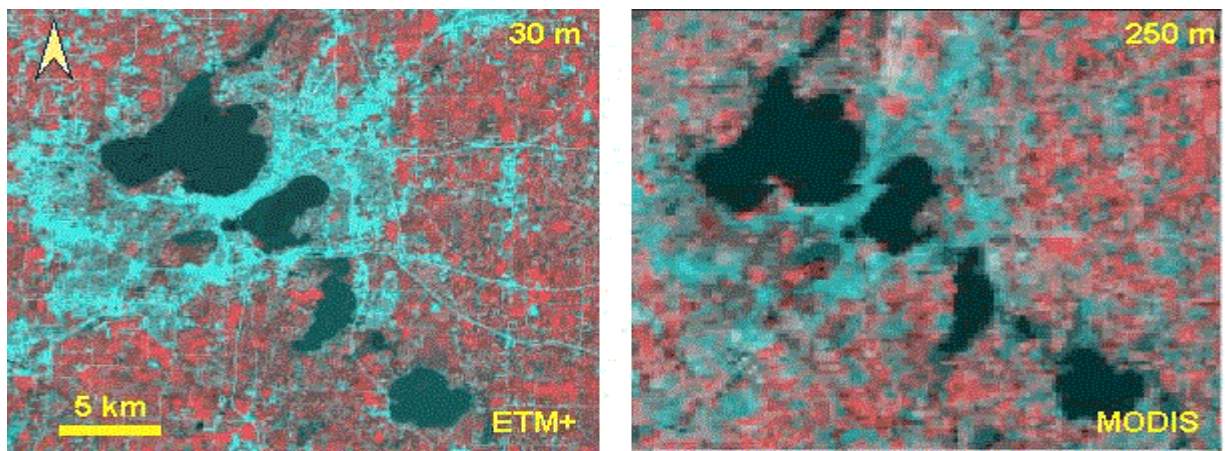


Fig. 2.Spatial Resolution comparison between Landsat-8 and MODIS Image

Linear unmixing has rapidly become one of the most popular techniques to determine the content of a remotely sensed spectral image. There are techniques of spectral unmixing to extract independent features located at given area of interest on earth surface on the basis of its reflected radiance in the form of percentage parameter so to estimates its abundance (urban, water, vegetation etc). Its concept is established on the idea that each captured pixel in a spectral image, which is composed of N_b spectral bands, can be represented as a linear combination of a set of p spectrally pure constituent spectra or endmembers e , weighted by an abundance factor a , which denotes the proportion of each endmember in the pixel under study. Linear mixture doesn't consider the account of secondary reflections and scattering effects.

More importantly Linear mixture is based on the non-negativity (>0) and sum to one (0.25(A) + 0.25(B) + 0.50(C) = 1) factor of the fraction materials present in the single pixel or FOV(Field of view) which has been shown in the Fig. 3.

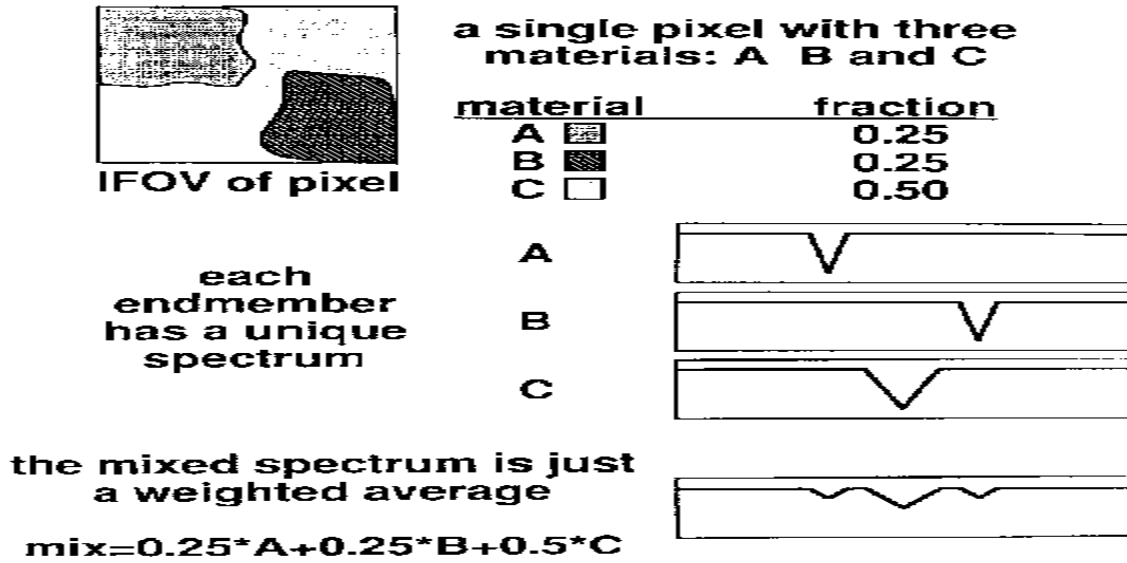


Fig. 3. Linear Mixture Model

2. Algorithms Used

2.1 Linear Mixture Model

Linear mixture model (Fig.3) states that the pixel data in any image band can be treated as a linear combination of the spectral response of component within the pixel with the prior knowledge of the endmember spectrum. The relation can be modeled as a set of vector matrix equation.

$$P_i = \sum_{j=1}^n (R_{ij} * F_{ij}) + E_i \dots \dots \dots (eq1)$$

Where,

- $i = 1, \dots, m$ (number of bands);
- $j = 1, \dots, n$ (number of endmembers);
- P_i = spectral reflectance of the i^{th} spectral band of a pixel;
- R_{ij} = known spectral reflectance of the j^{th} component;
- F_i = the fraction coefficient of the j^{th} component within the pixel;
- E_i = error for the i^{th} spectral band;

The error E is generated on the factor of non comparable reflectance and noise from the image. Applying the equation (1) on every spectral band it dissolves into matrix form of the linear unmixing equations. For the equation to be valid in real world conditions than the sum of the abundances should be one ($\sum_{j=1}^n F_j = 1$) and it be non-negative ($F_j \geq 0$).

2.2 Least Square Classification

Least square is a standard approach in regression analysis to the approximate solution of over-determined system. Overall solution minimizes the sum of the square of the errors. It follows the curve -fitting technique where the model is allowed to follow the line or curve until it finds the best fit.

Least fitted equation for classification is given by,

$$\hat{F} = (R^T R)^{-1} R^T P$$

2.3 Fuzzy Classification

2.3.1 What is fuzzy logic?

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) **Boolean logic** on which the modern computer is based. The idea of **fuzzy logic** was first reported by Dr. Lotfi Zadeh of the University of California at Berkeley in the 1960s.

The **Membership Function** of a fuzzy set is a generalization of the indicator function in classical sets. In fuzzy logic, it represents the degree of truth as a method of evaluation and membership function (with a range covering the interval (0,1) operating on the domain of all possible values.

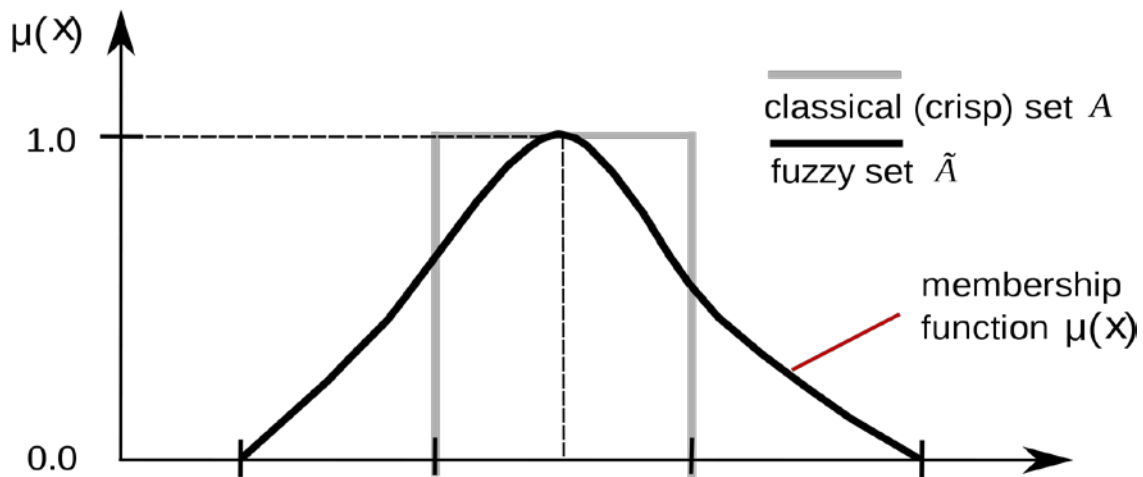


Fig. 4. Membership Function

2.3.2 Fuzzy Clustering Techniques for Unmixing (FCM)

It is a data clustering techniques in which the data set is grouped into n -clusters. Data point located close to the center of cluster will have a high degree of belonging or membership value.

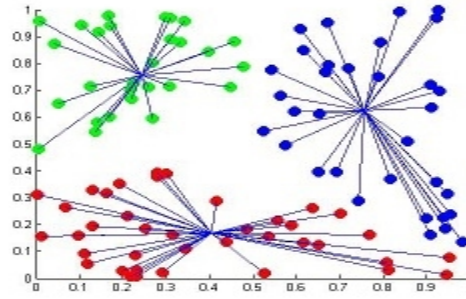


Fig. 5 Fuzzy Clustering (FCM)

The process can be described as:-

The clustering algorithm (FCM) that has been commonly adapted for supervised classification of remotely sensed data .The algorithm is based on the following function.

$$J_m = \sum_{i=1}^N \sum_{j=1}^C u_{ij}^m \|x_i - c_j\|^2, \quad 1 \leq m < \infty \dots\dots\dots(2)$$

where m is any real number greater than 1, N is the number of data, C is the number of cluster, u_{ij} is the degree of membership of x_i in the cluster j , x_i is the i th of d -dimensional measured data , c_j is the d -dimension center of the cluster.

2.4 Fast Linearly Unmixing

This one falls in the category of least square unmixing although maintaining the same accuracy the main advantage is that it takes a way too less time for computation process thus conserving time and energy.

The Fast algorithm uses modified Gram-Schmidt method. The operations performed are simple and can be easily parallelized. Abundances are calculated by spanning the endmembers component into space of orthogonal projection.

2.4.1 Gram Schmidt Orthogonalization

Gram-Schmid method calculates the orthogonal projection of a vector e_i on a set of vectors $E=[e_1,e_2,\dots,e_k]$ containing endmembers matrix with $k < i$ by subtracting the portion of the vector e_i contained in the directions spanned by the vectors $E=[e_1,e_2,\dots,e_k]$ with $k < i$ (3).After processing on a set of independent vectors $E=[e_1,e_2,\dots,e_n]$ it yields a set of orthogonal vectors $Q=[q_1,q_2,\dots,q_n]$ and its normalized vectors $U=[u_1,u_2,\dots,u_n]$

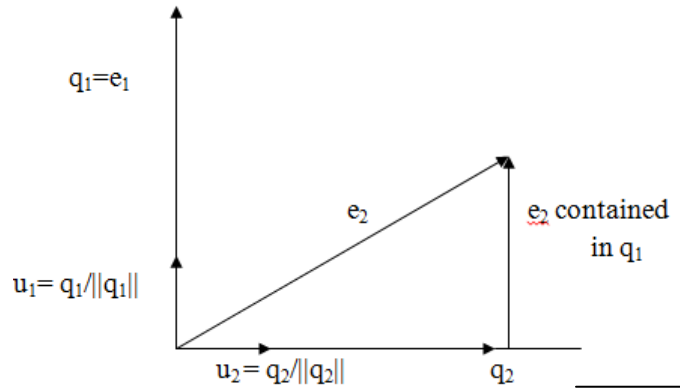


Fig. 6. Gram Schmidt orthogonalization of the endmember e2 with respect to endmember e1

It follows the same concept of the Linear unmixing model.

2.5 Distance Geometry Based Linear Unmixing

For linear unmixing it uses the concept of convex geometry in which the endmembers are spread in the space such a way that their position are assumed to be as vertices of simplex surrounding the spectral image data and the barycentric coordinates of the observation pixels with respect to the simplex related to the abundance of endmembers. By obtaining the relationship based on distance concept abundance map can be constructed

In this unmixing method we use Cayley-Menger (4) matrix which makes calculating barycentric coordinates more easily for the observed pixels. Apart from these optimal estimated points of observation pixels as well as the least distortion in geometric structure can be obtained. Its main limitation is that accuracy gradually decreases with the increasing number of endmembers.

2.5.1 Linear Mixture Model

$$P_i = \sum_{j=0}^n (P_{ij} * F_{ij}) + E_i$$

Already discussed in **section 2.1**

2.5.2 Convex Geometry for spectral data

Data point of the spectral data should lie in a simplex spanned by endmembers according to convex geometry theory. Affine hull covered by endmembers called as affine endmember-hull as shown in Fig. 13.

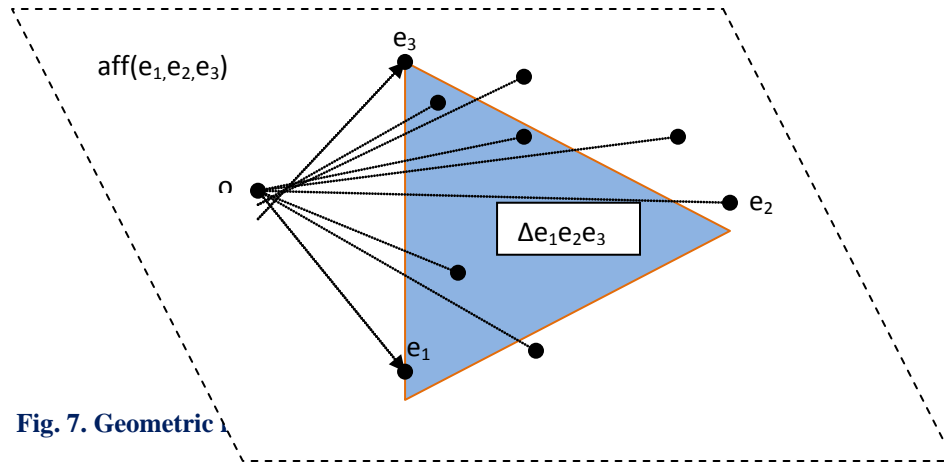


Fig. 7. Geometric

3. Results Comparison

3.1 Test Image

It is the subset of the Uttarakhand area from Landsat -7(ETM+) data taken on May 2003 representing little part of the Garhwal area consisting mostly of water, vegetation and hilly areas. The image has the size of 682 rows by 856 columns and 6 bands.



Fig 8. Test Image of Data 2(FCC)

3.2 Comparisons Assessment of the following techniques.

For comparing among the various unmixing algorithms results have been taken a single class as water. Here are the following unmixing results.

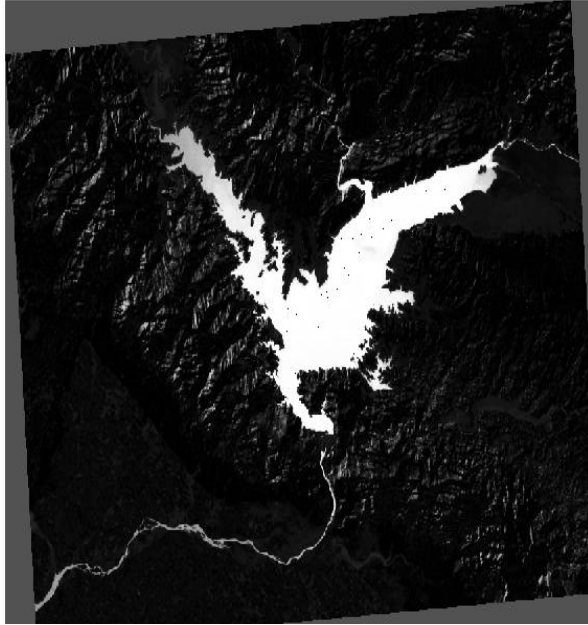


Fig 9. Fuzzy Unmixing



Fig 10. Fast Unmixing

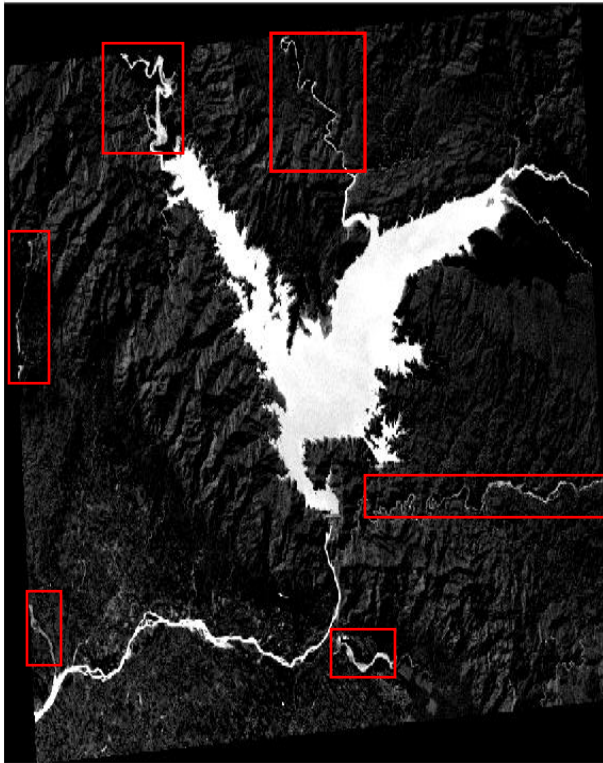


Fig 11. Distance Based Unmixing

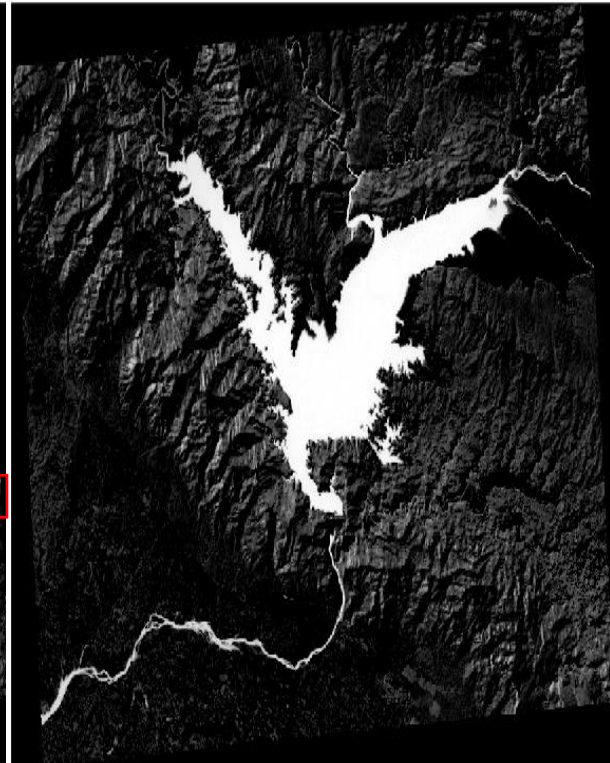


Fig 12. Least Square Unmixing

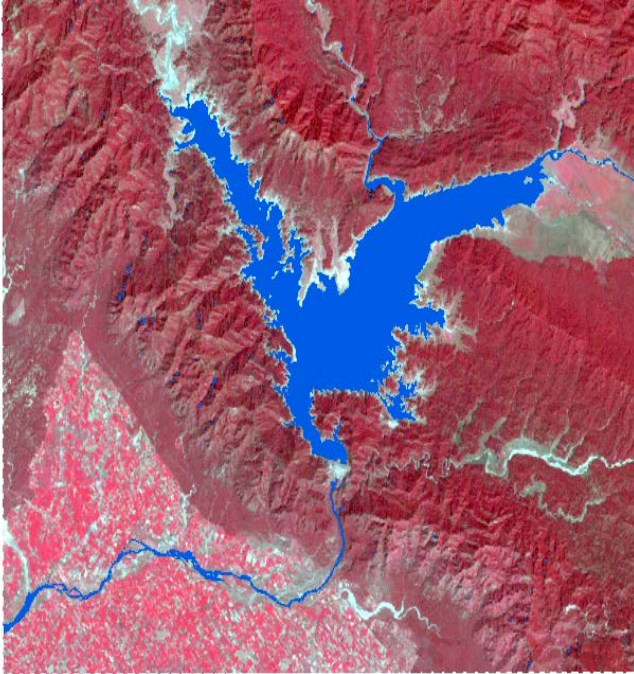


Fig 13. Software Based Unsupervised Unmixing (ArcGis)

As talking about visual comparisons the Distance based unmixing provides the better results with respect to others techniques even the present unmixing software fails to provide that level of details more over distance unmixing covers the minute tributaries of water areas more uniformly which has been highlighted by red box and level of accuracy is high among others.

In terms of quantitative analysis area covered by class by unmixing algorithms are:-

Table 1. Quantitative analysis of unmixing algorithms

Algorithm	Class (Water)	Area Covered (meter square)
Fuzzy		30960900
Distance		39638700
FCLS		34788600
Fast		34788600

From the Table 1it is observed that Distance Unmxing had covered more of the area followed by FCIS, Fast Unmixing and the least is the Fuzzy unmixing. Software based unmixing if considered it takes less execution time and Distance followed by Fuzzy, Fast and FCLS.So the Distance Based Unmxing not only provides better result but also takes less time.

4.0 Conclusions

Spectral mixture analysis provides an efficient mechanism for the interpretation of remotely sensed multidimensional imagery. It aims on the basis of a set of reference signatures (endmembers) that can be used to model the reflectance spectrum at each pixel of the original image. Through this research attempt is made to compare various unmixing techniques for generating abundance maps in the search of better results in aspects of accuracy, efficiency and lesser execution time. Distance based unmixing method proved to be an effective technique in classifying a water resources in remote sensed image followed by the Fully Constrained Least Square and Fast Unmixing and the last is Fuzzy Unmixing.

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References

- [1] Yi-Hsing TSENG, "*Spectral Unmixing for the Classification of Hyperspectral Images*".
- [2] Giles M. Foody, "*Estimation of Sub-pixel Land Cover Composition in the presence of Untrained Classes*" Computers & Geosciences 26 (2000) pp. 469,478
- [3] Raul Guerra, Lucana Santos, Sebastian Lopez, Senior Member, IEEE, and Roberto Sarmiento, "*A New Fast Algorithm for Linearly Unmixing Hyperspectral Images*".
- [4] Hanye Pu, Wei Xia, Student, IEEE, Bin Wang, Senior Member, IEEE, and Geng-Ming Jiang, "*A Fully Constrained Linear Spectral Unmixing Algorithm Based on Distance Geometry*".