THE UNCERTAINTY ESTIMATION OF INTERIOR AND EXTERIOR ORIENTATION PARAMETERS BASED ON VANISHING POINT MEASUREMENTS

Wei-Tong Chen [1] and Jen-Jer Jaw [2]

[1,2] National Taiwan University (NTU), No. 1, Sec. 4, Roosevelt Rd., Taipei 10617, Taiwan (R.O.C.),

Email: r03521114@ntu.edu.tw[1], jejaw@ntu.edu.tw[2]

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ABSTRACT： In perspective projection, all parallel lines in object space are imaged to converge towards a spot called vanishing point. The position and distribution of vanishing points imply the pose correspondence between object space and image space. The estimation of interior and exterior orientation parameters using vanishing points is based on some assumptions in object space and image plane. For instance, the lines intersecting to form vanishing point must be parallel in object space and those parallel line pairs used to determine poses need to be mutually orthogonal. However, owing to the fact that every measurement is subject to some uncertainty together with the reality of object space geometry, fulfilling these ideal assumptions seems to be unrealistic and the performance of estimating orientation parameter must be affected as a consequence. The purpose of this study is to discuss the impact of quality and distribution of vanishing points on the estimation of orientation parameters. Furthermore, an efficient way to assess the orientation parameters concerning different scene geometry, image source type, and vanishing point measurements has been established as the result of this work.

1. INTRODUCTION

In perspective projection, all parallel lines in object space are imaged to converge towards a spot called vanishing point as illustrated in Figure 1. Vanishing points imply the pose correspondence between object space and image space and can be used to get three interior parameters (x_0, y_0, f) and three exterior direction parameters $(\varphi, \omega, \kappa)$ （Li, 2012）. Therefore, the related researches have focused on camera calibration and pose estimation. However, the estimation of interior and exterior orientation parameters using vanishing points is based on some assumptions in object space and image plane. For instance, the lines intersecting to form vanishing point must be parallel in object space and those parallel line pairs used to determine poses need to be mutually orthogonal.

Figure 1. Vanishing point

Owing to the fact that every measurement is subject to some uncertainty together with the reality of object space geometry, fulfilling these ideal assumptions seems to be unrealistic and the performance of estimating orientation parameter must be affected as a consequence. It is essential to estimate the uncertainty of vanishing points resulting from such non-ideal factors and fairly quantify the amount of errors on the estimated parameters.

2. MATHEMATIC BACKGROUND

Suppose vanishing points which converge by parallel lines in three orthogonal directions respectively are X_∞, Y_∞ and Z_∞. The coordinates of vanishing points can be derived from collinearity equation：

$$x_a = x_0 - f\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} \quad (1)$$

$$\mathrm{y_a} = \mathrm{y_0} - f\frac{\mathrm{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}}{\mathrm{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}} \quad (2)$$

In Eqs.（1）and（2）, x_a and $\mathrm{y_a}$ are the photo coordinates of image point a; X_A, Y_A and $\mathrm{Z_A}$ are object space coordinates of point A; f is camera focal length; $x_0, \mathrm{y_0}$ are coordinates of principal point; and the $\mathrm{m's}$ （as described in Eqs.（3）to（11）) are functions of rotation angles which rotated in $\omega - \varphi - \kappa$ sequentially.

$$m_{11} = \cos\varphi\cos\kappa \quad (3)$$
$$m_{12} = \sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa \quad (4)$$
$$m_{13} = -\cos\omega\sin\varphi\cos\kappa + \sin\omega\sin\kappa \quad (5)$$
$$m_{21} = -\cos\varphi\sin\kappa \quad (6)$$
$$m_{22} = -\sin\omega\sin\varphi\sin\kappa + \cos\omega\cos\kappa \quad (7)$$
$$m_{23} = \cos\omega\sin\varphi\sin\kappa + \sin\omega\cos\kappa \quad (8)$$
$$m_{31} = \sin\varphi \quad (9)$$
$$m_{32} = -\sin\omega\cos\varphi \quad (10)$$
$$m_{33} = \cos\omega\cos\varphi \quad (11)$$

Let x coordinates of object space point X_A approaches infinity, numerator and denominator of rational expressions in Eqs.（1）and （2）divided by X_A. Get vanishing point coordinates of X_∞ which converge by parallel lines in X direction (Rong，2007):

$$x_{X_\infty} = x_0 - f\frac{m_{11}}{m_{31}} \quad (12) \text{ (Rong，2007)}$$

$$y_{X_\infty} = y_0 - f\frac{m_{21}}{m_{31}} \quad (13) \text{ (Rong，2007)}$$

Similarly, vanishing point coordinates of Y_∞ and Z_∞ are:

$$x_{Y_\infty} = x_0 - f\frac{m_{12}}{m_{32}} \quad (14) \quad (\text{Rong, 2007})$$

$$y_{Y_\infty} = y_0 - f\frac{m_{22}}{m_{32}} \quad (15) \quad (\text{Rong, 2007})$$

$$x_{Z_\infty} = x_0 - f\frac{m_{13}}{m_{33}} \quad (16) \quad (\text{Rong, 2007})$$

$$y_{Z_\infty} = y_0 - f\frac{m_{23}}{m_{33}} \quad (17) \quad (\text{Rong, 2007})$$

From Eqs. (12) to (17) we can learn that vanishing points are function of interior and exterior orientation parameters. Besides, geometry of oblique image as shown in Figure 2, where L denotes the perspective center and xyz is image coordinate system. The rotation about X axis through an angle ω, the rotation about Y axis through an angle φ and the rotation about Z axis through an angle κ. Rotation angle ω also can be expressed as the angle between line $\overline{Lo}$ (o is the principal point) projects on YZ plane and plumb line through the perspective center, angle φ is the angle between line $\overline{Lo}$ and its projection on YZ plane and κ is the angle between the projection of X axis in object space coordinate system on image plane and x axis in image coordinate system(Wang, 2011).

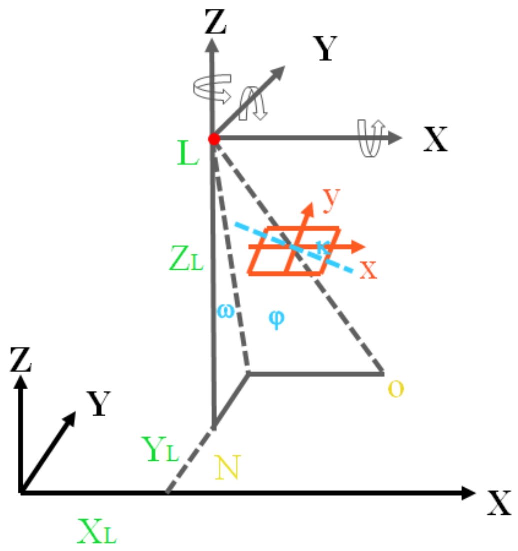

Figure 2. Geometry of oblique image (Wang, 2011)

Take κ for example. Due to lines parallel to X axis in object space coordinate system converge to vanishing point X_∞, κ can be derived from coordinates of X_∞:

$$\tan\kappa = \frac{-(y_{X_\infty} - y_0)}{(x_{X_\infty} - x_0)} \qquad (18)$$

According to Eqs. (18) , it reveals the relationship between vanishing point X_∞ and attitude angle κ. The geometric relation relationship between vanishing points and orientation parameters as shown in Figure 3. Owing to the fact that the line joining the vanishing point and the perspective center is parallel to the lines which compose the very vanishing point（Chen, 2003）, L-$X_\infty Y_\infty Z_\infty$ is a rectangular tetrahedron and the principal point o is the orthocenter of $\Delta X_\infty Y_\infty Z_\infty$. Depending on Figure 3, we can derive the relations between vanishing points and orientation parameters：

$$tan\,\omega = \frac{\sqrt{f^2 + (x_{Z_\infty} - x_0)^2 + (y_{Z_\infty} - y_0)^2}}{\sqrt{f^2 + (x_{Y_\infty} - x_0)^2 + (y_{Y_\infty} - y_0)^2}} \quad (19)$$

$$tan\,\phi = \frac{f}{\sqrt{(x_{X_\infty} - x_0)^2 + (y_{X_\infty} - y_0)^2}} \quad (20)$$

$$f = \sqrt{-(x_{X_\infty} - x_0)\cdot(x_{Z_\infty} - x_0) - (y_{X_\infty} - y_0)\cdot(y_{Z_\infty} - y_0)} \quad (21)$$

$$= \sqrt{-(x_{X_\infty} - x_0)\cdot(x_{Y_\infty} - x_0) - (y_{X_\infty} - y_0)\cdot(y_{Y_\infty} - y_0)} \quad (22)$$

$$= \sqrt{-(x_{Y_\infty} - x_0)\cdot(x_{Z_\infty} - x_0) - (y_{Y_\infty} - y_0)\cdot(y_{Z_\infty} - y_0)} \quad (23)$$

$$x_0 = \frac{\begin{vmatrix} x_{Y_\infty}x_{Z_\infty} + y_{Y_\infty}y_{Z_\infty} & y_{X_\infty} & 1 \\ x_{X_\infty}x_{Z_\infty} + y_{X_\infty}y_{Z_\infty} & y_{Y_\infty} & 1 \\ x_{X_\infty}x_{Y_\infty} + y_{X_\infty}y_{Y_\infty} & y_{Z_\infty} & 1 \end{vmatrix}}{2\Delta} \quad (24)(Chou，2014)$$

$$y_0 = \frac{\begin{vmatrix} x_{X_\infty} & x_{Y_\infty}x_{Z_\infty} + y_{Y_\infty}y_{Z_\infty} & 1 \\ x_{Y_\infty} & x_{X_\infty}x_{Z_\infty} + y_{X_\infty}y_{Z_\infty} & 1 \\ x_{Z_\infty} & x_{X_\infty}x_{Y_\infty} + y_{X_\infty}y_{Y_\infty} & 1 \end{vmatrix}}{2\Delta} \quad (25)(Chou，2014)$$

$$\Delta = \frac{1}{2}\begin{vmatrix} x_{X_\infty} & y_{X_\infty} & 1 \\ x_{Y_\infty} & y_{Y_\infty} & 1 \\ x_{Z_\infty} & y_{Z_\infty} & 1 \end{vmatrix} \quad (26)(Chou，2014)$$

The line which parallels to object space parallel lines and passes through perspective center intersects image plane at vanishing point（Chang, 1982）. Therefore, the direction of ω and φ (positive or negative) have impact on direction of dip which can decide the positions of vanishing points.

Since rotation angle κ is more intuitively clear, the following section 3 and 4 take the example of κ. In section 3, model the estimation of theoretical precision of κ with error propagation. Moreover, simulate error under the condition of non-ideal assumption of object space geometry which impact on quality of attitude angle κ. In section 4, generate object point with error in Y coordinate result in non-parallel lines which affect coordinates and quality of vanishing point.

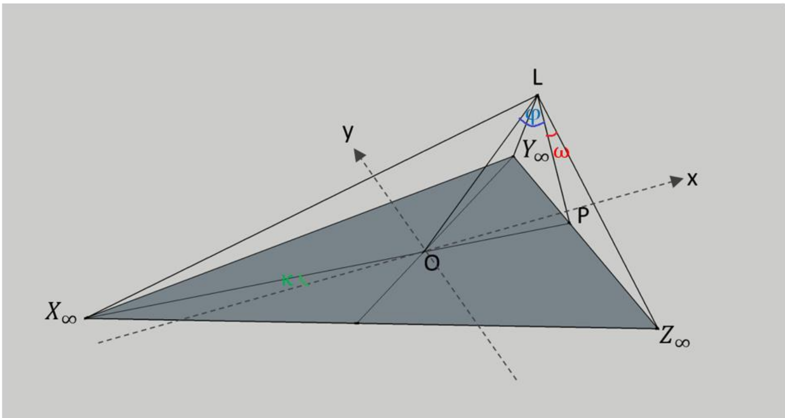

Figure 3. Geometry of rotation angles and vanishing points

3. ERROR ESTIMATION OF POSE ANGE κ

Supposed principal point (x_0, y_0) is errorless, coordinates of vanishing point X_∞ $(x_{X_\infty}, y_{X_\infty})$ are observations in Eqs.（18）. According to error propagation law, the accuracy of attitude $\tan\boldsymbol{\kappa}$ is estimated by:

$$\Sigma_{\tan\boldsymbol{\kappa}\tan\boldsymbol{\kappa}} = J_{\tan\boldsymbol{\kappa}x} \cdot \Sigma_{xx} \cdot J_{\tan\boldsymbol{\kappa}x}{}^{T} \qquad (27)$$

Where Σ_{xx} is co-factor matrix and $J_{\tan\kappa x}$ is Jacobian matrix.

$$\Sigma_{xx} = \begin{bmatrix} \sigma_{x_{X_\infty}}{}^2 & \sigma_{x_{X_\infty} y_{X_\infty}} \\ \sigma_{y_{X_\infty} x_{X_\infty}} & \sigma_{y_{X_\infty}}{}^2 \end{bmatrix} \qquad (28)$$

$$J_{\tan\kappa x} = \begin{bmatrix} \dfrac{\partial \tan\kappa}{\partial x_{X_\infty}} & \dfrac{\partial \tan\kappa}{\partial y_{X_\infty}} \end{bmatrix} = \begin{bmatrix} \dfrac{(y_{X_\infty} - y_0)}{(x_{X_\infty} - x_0)^2} & \dfrac{-1}{(x_{X_\infty} - x_0)} \end{bmatrix} \qquad (29)$$

Substitute Eqs.（28）and（29）to（27）. Rewrite Eqs.（27）to:

$$\Sigma_{\tan\boldsymbol{\kappa}\tan\boldsymbol{\kappa}} = \begin{bmatrix} \dfrac{(y_{X_\infty} - y_0)}{(x_{X_\infty} - x_0)^2} & \dfrac{-1}{(x_{X_\infty} - x_0)} \end{bmatrix} \begin{bmatrix} \sigma_{x_{X_\infty}}{}^2 & \sigma_{x_{X_\infty} y_{X_\infty}} \\ \sigma_{y_{X_\infty} x_{X_\infty}} & \sigma_{y_{X_\infty}}{}^2 \end{bmatrix} \begin{bmatrix} \dfrac{(y_{X_\infty} - y_0)}{(x_{X_\infty} - x_0)^2} \\ \dfrac{-1}{(x_{X_\infty} - x_0)} \end{bmatrix}$$

$$= \frac{(y_{X_\infty} - y_0)^2 \sigma_{x_{X_\infty}}{}^2}{(x_{X_\infty} - x_0)^4} + \frac{\sigma_{y_{X_\infty}}{}^2}{(x_{X_\infty} - x_0)^2} - 2\frac{(y_{X_\infty} - y_0)\sigma_{y_{X_\infty} x_{X_\infty}}}{(x_{X_\infty} - x_0)^3} \qquad (30)$$

Because κ is inverse of $\tan\kappa$:

$$\kappa = \tan^{-1}\left(\frac{-(y_{X_\infty} - y_0)}{(x_{X_\infty} - x_0)}\right) \qquad (31)$$

Then, error in κ which derive from vanishing point is:

$$\sigma_{\kappa}{}^{2} = \Sigma_{\kappa\kappa} = J_{\kappa \tan\kappa} \cdot \Sigma_{\tan\kappa \tan\kappa} \cdot J_{\kappa \tan\kappa}{}^{T} = \frac{1}{(tan^{2}\kappa + 1)^{2}} \cdot \Sigma_{\tan\kappa \tan\kappa}$$

$$= \frac{sin^{2}\kappa}{\left(x_{X_{\infty}} - x_{0}\right)^{2}} \sigma_{x_{X_{\infty}}}{}^{2} + \frac{cos^{2}\kappa}{\left(x_{X_{\infty}} - x_{0}\right)^{2}} \sigma_{y_{X_{\infty}}}{}^{2} + 2\frac{sin\kappa \cdot cos\kappa}{\left(x_{X_{\infty}} - x_{0}\right)^{2}} \sigma_{y_{X_{\infty}} x_{X_{\infty}}} \qquad (32)$$

According to Eqs. (32), error in κ is related to the position and quality of vanishing point X_{∞}. The distance between vanishing point X_{∞} and principal point in x direction is negative related to accuracy of κ. When the value of κ increase, $sin\kappa$ increase positively; nevertheless $cos\kappa$ decreased negatively. At the same time, pose κ and the position of vanishing point react upon each other.

4. ERROR IN NON_IDEAL ASSUMPTION OF OBJECT SPACE GEOMETRY

To simulate error under the condition of non-ideal assumption of object space geometry, we generate object point with error in Y coordinate to make parallel lines non-parallel. There are four object space points 1~4 and Figure 4 shows the distribution of object points. To generate non parallel line, vary y-coordinate of object point 3 from 1700 meter to 1695 meter and the interval is 1 meter. With error in object point 3, make blue line shifting to red line in Figure 4. The parallel lines consist of $\overline{12}$ and $\overline{34}$ become non-parallel.

Project object space points 1~4 to image space with Eqs. (1) and (2) and the parameter setting is illustrated in Table 1 .The two lines on image plane intersect at a point so called vanishing point while two lines are parallel. The result is shown in Table 2. The position of vanishing point varies with error in assumption of object space geometry which demonstrates that non-ideal assumption of geometry has impact on accuracy of vanishing point and quality of following applications.

Table 1. Parameter setting of experiment

Parameter Setting			
ω, φ, κ	$(3°, 4°, 5°)$	**Object point 1**	$(1500m, 1500m, 100m)$
Perspective center	(1150m,1150m,1500m)	**Object point 2**	$(1700m, 1500m, 100m)$
Image size	$23cm \times 23\ cm$	**Object point 3**	$(1500m, 1700m, 100m)$
Scale	$1{:}\,10000$	**Object point 4**	$(1700m, 1700m, 100m)$
Focal length	$15\ cm$	**Error in y coordinate of object point 3**	$1{\sim}5m$

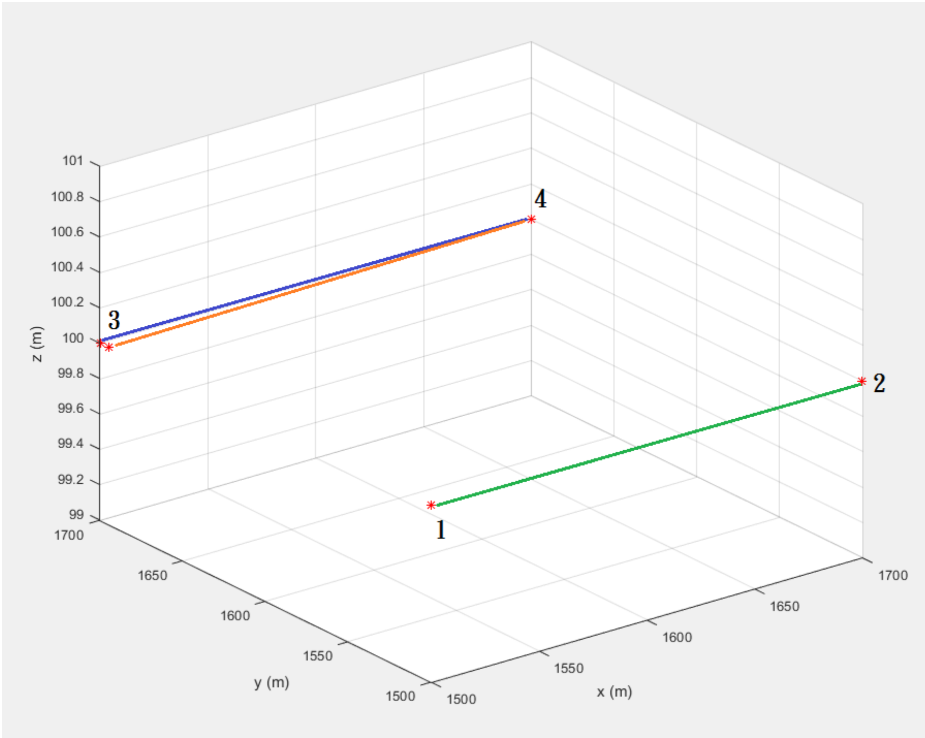

Figure 4. Distribution of object space points

Table 2. Vanishing point with error in assumption of object space geometry

Error in y coordinate of object point 3 (unit: m)	Error of parallel line in object space (unit: deg)	Vanishing point x coordinate (unit: m)	Vanishing point y coordinate (unit: m)
0	0	-2.13693719	0.18695778
1	0.28647651	-1.40755575	0.13313253
2	0.5729387	-1.04015961	0.10602026
3	0.85937224	-0.81884847	0.08968844
4	1.14576284	-0.67094196	0.07877357
5	1.43209618	-0.56511475	0.07096398

With Eqs.（31）, rotation angle κ can be calculated and the result as shown in Table 3. According to Table 3, The larger error in assumption of object space geometry, the lower accuracy of rotation angle κ.

Table 3. Rotation angle κ error under the condition of non-parallel line in object space

Error in y coordinate of object point 3 (unit: m)	Error of parallel line in object space (unit: deg)	k derived from vanishing point (unit: deg)	Error of k (unit: deg)
0	0	5	0
1	0.28647651	5.40320107	0.40320107
2	0.5729387	5.81988334	0.81988334
3	0.85937224	6.25068808	1.25068808
4	1.14576284	6.69629433	1.69629433
5	1.43209618	7.15742148	2.15742148

5. CONCLUSION

The estimation of interior and exterior orientation parameters using vanishing points is based on some assumptions in object space and image plane. Owing to the non-ideal condition on real world, errors in such assumptions make influence on the quality of derived pose angles. According to our experiment, the distance and quality of vanishing point, the value of exterior orientation and the assumption in object space geometry all impact on the uncertainty of exterior orientation. In this paper we take rotation angle κ as an example and we will keep working on modeling the uncertainty of rotation angles ω and φ and interior orienttaion parameters.

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