FILTERING OF SUBWAY TUNNELPOINT CLOUD BASEDON P-NORM MINIMUM FITTING METHOD

Zhang Lishuo[1] and Cheng Xiaojun[1]
1. College of Surveying and Geo-Informatics, Tongji University, 1239 Siping Road, Shanghai 200092, China.
Email: 2009302590086@whu.edu.cn

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ABSTRACT: With the improvement of the accuracy and efficiency of laser scanning technology, high-resolution terrestrial laser scanning (TLS) technology which can obtain high precise points-cloud has been applied to high-precision deformation monitoring of subway tunnels, building structures and other fields. Many problems in geometry processing are stated as least-squares (2-norm minimum) optimizations. Least-squares problems are well studied and widely used but exhibit immanent drawbacks such as high sensitivity to outliers Theoretical backing for using least-squares methods is given by the Gauss–Markov theorem which basically states, that if the input data to the problem fulfills some statistical properties, e.g. zero mean, unique variance and zero covariance, the least-squares way is the best way to go. However, as praxis has shown, the input data does not always meet these requirements. Real world data poses severe challenges to the least-squares approach. One particular problem is outliers, which naturally occur in various ways in physical measurement processes. The threshold to remove the outliers is 2× or 3× mean square error in many literatures, which is not reasonable Experiments show that if the threshold is too large many inliers will be removed, in contrast, if the threshold is too small many outliers will not be removed. It is essential to choose an appropriate threshold. In this case, a method for removing outliers of the tunnel is presented, In view of the special section characteristic of the subway tunnel and its long shape This method firstly extracts tunnel cross sections and fitting the points on the slices to an ellipse using p-norm minimum algorithm, finally an adaptive threshold selection method based fitting residuals is introduced to identify and remove the outliers whose residuals are more than the threshold The method described in this paper has been tested and verified by the experiment using the data of a Shanghai subway tunnel. Results show that the p-norm minimum fitting algorithm is more robust than least-squares algorithm. The threshold can distinguish the outliers and inliers well. The proposed method is rather simple and can be easily implemented.

1 INTRODUCTON

It is a challenging task to monitor the stability of a civil technical structure, already being addressed for a long time in many literatures. In recent years, terrestrial laser scanning (TLS) is applied for the acquisition of accurate and dense 3D datasets, because a high scanning speed and high accuracy offer laser scanners significant advantages compared to more traditional techniques. Researches are gradually focused on applying 3D laser scanner into tunnel deformation monitoring. Gosliga (Gosliga, 2006) acquired the cylindrical model of tunnel through the least squares fitting algorithm, and proposed the statistical analysis method to analyze the deformation data of the tunnel. Li (Li, 2015) analyzed the polar coordinates radius differences of fitting ellipses and gained the deformation of tunnel sections. Nuttens(Nuttens, 2014) developed a method for the ovalization monitoring of newly built circular train tunnels based on laser scanning. Walton (Walton, 2014) applied the comparison of section cloud point data and least-squares fitting ellipse that represented the section surface to analyze the deformation. A tunnel point cloud denoising algorithm based on centerline was proposed by Cheng (Cheng, 2014), the central axis of the subway tunnel was determined by bilateral projection, a given distance threshold was used to accomplish data filtering

According to literatures, some problems existed in the deformation monitoring of tunnels by using TLS: firstly, fitting the tunnel profile by the least-squares algorithm is interfered by outliers; secondly, the threshold based on mean square error is either too large or too small to remove the outliers effectively. Therefore, a tunnel point cloud filtering method is proposed in this paper.

2 SUBWAY TUNNELPOINT CLOUD FILTERING

2.1 Cross Sections Extraction

To extract cross sections from terrestrial point clouds, the central axis of the subway tunnel must be determined. The central axis can be used to determine the direction of a cross-sectional plane at any interval. Similar to the idea of the 2D projection, the 3D central axis of a tunnel is estimated in terms of the projections of the tunnel points on the XOY and the YOZ planes. For more details, the reader is referred to Kang(2014).

2.2P-norm Minimum Ellipse Fitting

The formula for an ellipse is:

$$ax^2 + bxy + cy^2 + dx + ey - f = 0 \quad (1)$$

Where:

$b^2 - 4ac \leq 0$

The origin of the coordinate system is the instrument center, thus $f \neq 0$.Then the formula for an ellipse can be written as:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey - 1 = 0 \quad (2)$$

Where:

$B^2 - 4AC \leq 0$

To fit a set of data with an ellipse, the formula for an ellipse can be generalized to:

$$v_i = Ax_i^2 + Bx_iy_i + Cy_i^2 + Dx_i + Ey_i - 1 \quad (3)$$

Where the set of data to fit to is:

$(x_1, y_1, x_2, y_2 \cdots, x_n, y_n)$

The set of equations can then be written in matrix form:

$$V = GX - L \quad (4)$$

Where:

$$V = \begin{bmatrix} v_1 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{bmatrix}, G = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^2 & x_iy_i & y_i^2 & x_i & y_i \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_ny_n & y_n^2 & x_n & y_n \end{bmatrix}, X = \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix}, L = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

In the many literatures the authors chose least squares (V^TV=min) as adjustment constraint. However, this choice affects the capability of fitting algorithm to deal with noise and outliers. We formulate the fitting problem as recovering ellipse parameters that maximizes the number of zero distances between the points and the fitting ellipse. This can be achieved by minimizing the 0-norm of the vector of error residuals. However, due to the problems caused by the high non-convexity of the 0-norm, we solve the ellipse fitting problems using p-norm, where $p \in (0,1]$, instead of the classical least squares algorithm(p=2).

2.3 Adaptive Threshold

In many literatures, the outliers were removed based on the error residuals, which compare with Mean Square Error of Unit Weight σ
Where:

$$\sigma = \sqrt{\frac{V^TV}{n-5}} \tag{5}$$

Many authors chose σ, 2σ, 3σ as threshold to distinguish a point is outlier or not. When the absolute value of error residual is greater than the threshold, this point is an outlier. Experiments show that if the threshold is too large many inliers will be removed, in contrast, if the threshold is too small many outliers will not be removed. It is essential to choose an appropriate threshold.

A large number of experiments show that if the absolute value of error residuals is sorted in ascending order, this sequence like a right angle (see Fig.1). The turning point can be chosen as threshold. Accordingly, an adaptive threshold is proposed in this paper:

Step1: Sorting the absolute value of error residuals in ascending order, then every point has a new coordinate (sequence number, the absolute value of residual).
Step2: Connecting the first point and the last point to be a line segment, and then calculating the distance from each point to the line segment.
Step3: Finding the point whose distance is the largest, its vertical coordinate is the threshold.

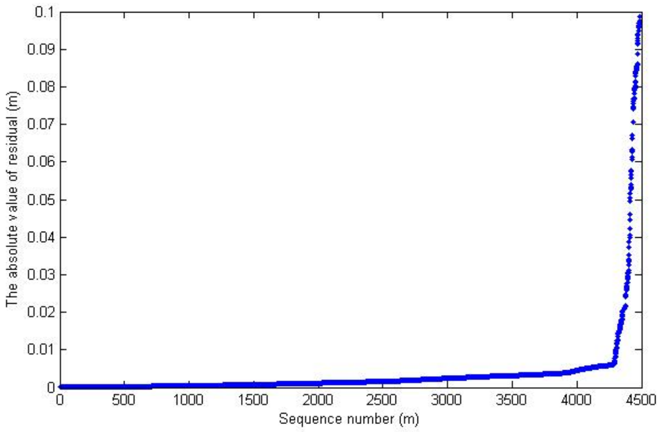


Figure 1 The Absolute Value of Residual.

3 EXPERIMENT AND ANALYSIS

The proposed approach was tested with a real dataset acquired by a laser scanner of FARO Focus3D X330 in a subway tunnel in Shanghai in different time periods. The sections are extracted from the raw scan data as shown in Figure 2. From Figure2 we can get there are many outliers in the sections which will affect the ellipse fitting.

Figure 2 Subway Tunnel Section Points.

3.1 Ellipse Fitting

In this paper, the ellipse fitting is fulfilled through the approach proposed in Section 2.2. Set p=0.5, p=1(1-norm minimum), p=2(classical least square algorithm) to fit the points of section to an ellipse. More experiments will be performed in the future.

In Figure 3, we plot section points, the fitted ellipse for different methods. In Figure 3, it is clear that the ellipse fitted by p-norm ($p \leqslant 1$) minimum fitting algorithm can fit the tunnel section points better than least squares algorithm. Least squares algorithm is sensitive to outliers.

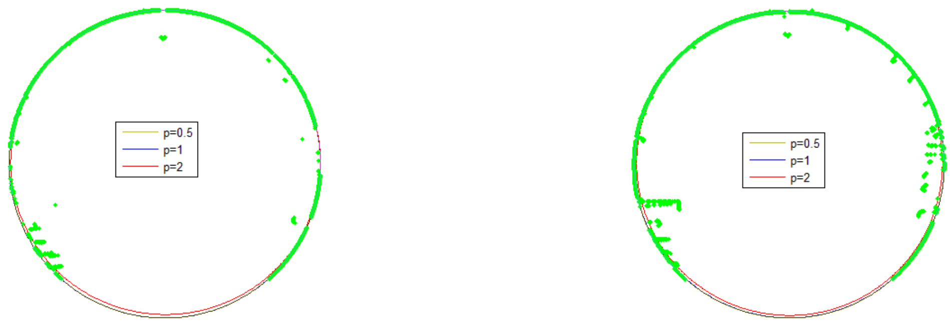


Figure 3 Sections Fitting (p=2, 1, 0.5).

3.2 Data Filtering Using Adaptive Threshold T

After fitting the ellipse using p=0.5, the residual victor is calculated using equation (4), and then, the threshold T was computed out through the approach proposed in section 2.3.σ of least squares ellipse fitting algorithm (p=2) Is calculated using equation (5) for contrast.

Figure 4 shows the section points (green), the ellipses fitted using p=2 and p=0.5. The data filtering results are given in Figure 5. From Figure 5 we can see, the threshold in this paper can distinguish the outliers and inliers well. Most outliers are removed through the approach proposed in this paper, however, using p=2, when T=σ, 2σ, many inliers were removed; when T=3σ, some outliers could not be removed. The principal reason is that least square algorithm has no capability to deal with noise and outliers.

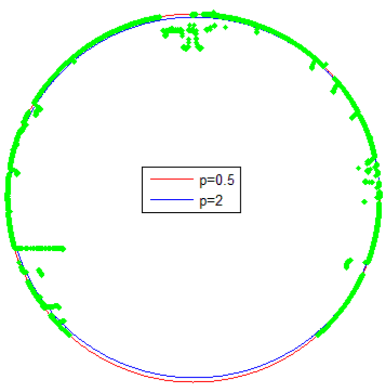
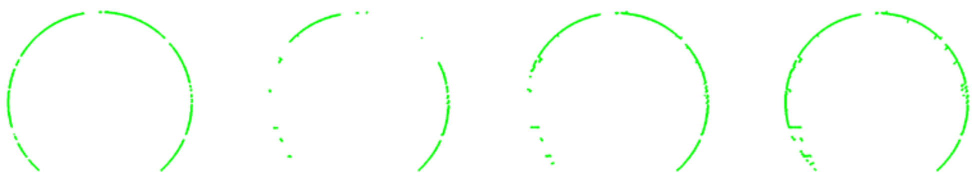


Figure 4 Section Fitting (p=2,0.5).



(a) T=t (b) T=σ (c) T=2σ (d) T=3σ

Figure 5 Data Filtering (t= 0.006m, σ=0.046m).

4 CONCLUSION

A new method for tunnel point cloud denoising method is proposed. The method described in this paper has been tested and verified by the experiment using the data of a subway tunnel in Shanghai. The ellipse experiments show that the p-norm($0<p\leq1$) minimum fitting algorithm is more robust than least-squares algorithm. And choose p between zero and one allows a tradeoff between efficiency and robustness, significantly improves the resilience of the method to large amounts of outliers in the point cloud. The adaptive threshold can effectively distinguish the outliers and inliers.

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