DERIVING SPATIAL RELATIONS OF BUILDING FEATURES USING CONDITIONAL DELAUNAY TRIANGULATION TO SUPPORT AUTOMATED MAP GENERALIZATION

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ABSTRACT: The paper first reviews the existing data enrichment techniques for automated map generalization based on several spatial data structures. Next, it provides a broad explanation of the well-known triangulation data structure found in the literature for enriching spatial relations of building polygons to be used in automated map generalization. This paper then presents a novel approach to enrich spatial relations for retrieving neighbourhood and proximity relations of building polygons using Delaunay triangulation, taking into account the edges of buildings as constraints based on a new principle called 'non-edge intersection' to support automated map generalization. This triangulation method is termed as conditional Delaunay triangulation (CnDT) since it respects empty-circumcircle criterion in generating triangles, enforcing edges of buildings as constraints in the triangulation process. The main significance of this triangulation method is the ability to derive explicit neighbourhood relations between building polygons without densification of edges of building geometries by adding new vertices into source data or interpolating artificial vertices on edges of buildings in the triangulation process. These neighbourhood relations enable extracting proximity relations efficiently direct from the object identification numbers tagged to three nodes of each triangle of the triangulation. The implementation of the approach is presented and described with several graphs including results to generate CnDT and the proximity relations of building polygonal features on the raw data set, ranging from simple geometries to complex geometries comprising of contiguous edges, polygon holes, and arbitrary split edges. Also, the results of the triangulation are compared with those obtained using the Constrained Delaunay triangulation method. Finally, the advantages and disadvantages of the proposed algorithm are discussed against the existing triangulation algorithms used for subsequent automated map generalization process.

1. INTRODUCTION

With the advancement of Internet and telecommunications technologies, the visualization of geographical information is no longer limited to hardcopy maps. Automated map generalization is required for the interactive and dynamic visualization of on-demand maps on the web and mobile devices at different resolutions according to user requirements. Automated map generalization is a difficult and time-consuming process and involves handling complex algorithms depending on the desired visual representation of the map content. One of the reasons for this difficulty is the lack of auxiliary information derived from the spatial data structures that describe spatial relationships of data as currently implemented in commercial spatial databases and Geographic Information System (GIS) software in support of existing generalization algorithms. There are several generalization functions based on various algorithms to enhance the visual representation of cartographic maps. The selection of an appropriate algorithm according to the visualization requirements is a complex process due to the contextual nature of generalization and depends to a great extent on the availability of auxiliary information to describe the characteristics of data in the raw data set to assist the generalization process. According to literature, there are a number of spatial data structures that extract auxiliary information from raw data to support automated generalization including Delaunay triangulation (DeLucia and Black, 1987; Jones, Bundy and Ware, 1995; Li *et al.*, 2004; Yan, Weibel and Yang, 2008; Qi and Li, 2008; Jones and Mark, 1998), Voronoi diagrams (Basaraner and Seluck, 2004; Yan, Weibel and Yang, 2008), minimum spanning trees (Regnauld, 2001, 2005; Qi and Li, 2008), graph structures (Mackaness and Beard, 1993; Regnauld, 2003, 2005), skeletons (Bader and Weibel, 1997) and hierarchical partitioning schemes (Ruas, 1995). However, the most widely used and effective geometric data structure in computational geometry to represent two horizontal relationships - topology and proximity - is the Delaunay triangulation (Delaunay, 1934). It has been employed in other spatial data structures; Voronoi diagrams, graph structures, and hierarchical partitioning schemes already described above in deriving spatial relationships. When the triangulation is Delaunay, it has the property that no point of the point set ***P*** is inside the circumcircle of any triangle in the triangulation (Berg *et al.*, 2008; Biniaz and Dastghaibyfard, 2012). Further, several algorithms have been discussed for Delaunay triangulation from vector point data - incremental (Berg *et al.*, 2008; Hjelle and Dæhlen, 2006; Guibas, Knuth and Sharir, 1992; Mehlhorn and Meiser, 1993), step-by-step approach (Hjelle and Dæhlen, 2006), radial sweep (Mirante and Weingarten, 1982), divide-and-conquer (Dwyer, 1987), sweep-line (Borut, 2005; Domiter and Žalik, 2008) and circle-sweep (Biniaz and Dastghaibyfard, 2012). Apart from the incremental algorithm, all the other algorithms are static in the sense that all points in a point set ***P*** must be known and treated starting from the first step of the algorithm.

Constrained Delaunay triangulation (CDT) is the generalization of conventional Delaunay triangulation in order to constrain a set of planar edges **E** in a triangulation comprising of a set of planar points **P** (Hjelle and Dµhlen, 2006) including the end points of edges **E**, thus **E** is a subset of edges in the final triangulation Δ**(G)**. In the CDT algorithm, the requirement is that the constrained edges must serve as the edges of the triangulation Δ**(G)**. That is; the edges between points in **P** must not intersect the interior of any constrained edge **E** or any triangulated region such as a hole or a region outside the exterior boundary of Δ**(G)**. With this requirement, the definition of conventional Delaunay triangulation can be modified to come out with a new circumcircle criterion definition which is according to Hjelle and Dæhlen (2006) is "a constrained Delaunay triangulation Δ**(G)** of a planar Points and Edges **G(P, E)** also known as planar straight line graph (PSLG), is a triangulation containing the edges **E** such that the circumcircle of any triangle **t** in Δ**(G)** contains no point of **P** in its interior which is visible from all the three nodes of **t**" (Figure 1(d)).

In this work, the conventional Delaunay triangulation, comprising of planar points **P** with planar edges **E** served as the same constraint edges as in the CDT, is termed as conditional Delaunay triangulation (CnDT) in which triangles are maintained as equilateral as possible, adopting the Delaunay property as discussed by Berg *et al.* (2008) and Biniaz and Dastghaibyfard (2012) above.

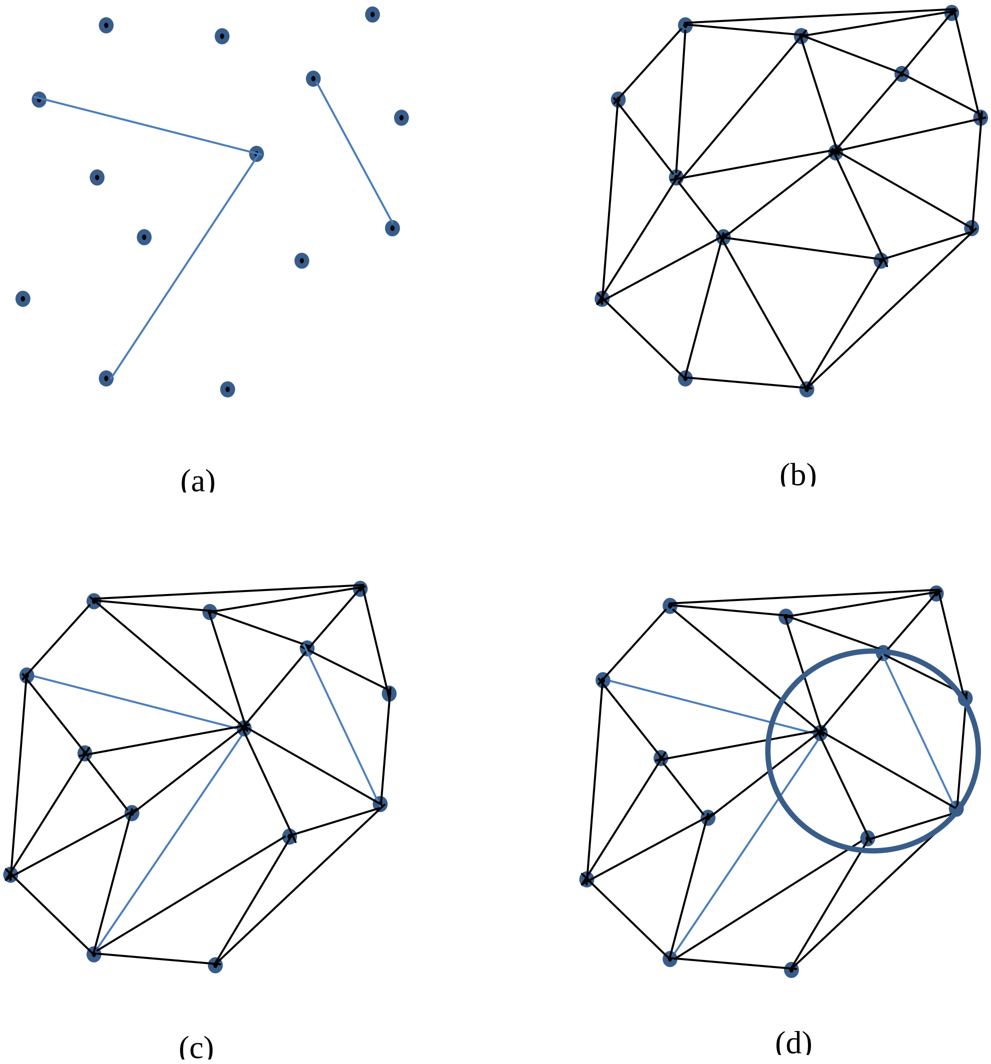


Figure 1 (a) A planar points and straight line edge graph G(P, E) called PSLG (b) conventional Delaunay triangulation of point set **P** (c) CDT of G(P, E) and (d) illustration of the modified circumcircle criterion on the hatched triangle for the CDT, based on Hjelle and Dæhlen (2006).

The process of adding auxiliary information to a raw data set is called the data enrichment. Current data enrichment methods focus on two main relations: horizontal and vertical as identified by (Neun, Weibel and Burghardt, 2004). Horizontal relations exist in the same level of detail (LOD) in a data set and represent common structural properties such as proximity, topology, pattern and alignment and helps recognize and derive clusters of geometrical objects, while vertical relations can exist among homologous objects or group of objects in different resolutions in terms of similarity of shape, size, compactness, links between object identification numbers (IDNs) and scale relations. Identification of horizontal relations in a data set not only helps identify structural knowledge required to deal with the contextual nature of map generalization but also enables leverage data sets at different resolutions in vertical relation to represent the same phenomena. These vertical relations can exist on both attributes and geometric features at these different resolutions and are important to leverage the structural knowledge (horizontal relations) of different LODs. In vertical relations, link details such as object IDNs of different data sets can be integrated and maintained either in one separate table or among the tables of the data sets in a Multiple Database Management System (MDBMS) (Hampe, Anders and Sester, 2003).

The aim of this paper is to introduce a new triangulation approach based on Delaunay triangulation, already introduced as CnDT, to enrich horizontal relations by extracting contextual, topological and/or proximity of complex polygonal building features, and to gain an insight into processing efficiency of such enriching information for the use of subsequent building generalization process.

The remainder of this paper is organized as follows: Section 2 discusses the background to data enrichment of building features for automated map generalization using Delaunay triangulation. Then the algorithm used to generate conditional Delaunay triangulation (CnDT) is described in Section 3. Section 4 describes the implementation of the proposed approach. Results of the proposed CnDT and its comparison between the Constrained Delaunay triangulation (CDT) are discussed in Section 5. Section 6 provides a detailed discussion about the significance of the proposed CnDT. Finally, conclusions are drawn in Section 7.

2. BACKGROUND ON DATA ENRICHMENT OF BUILDINGS WITH DELAUNAY TRIANGULATION

Jones, Bundy and Ware (1995) have used constrained Delaunay triangulation (CDT) in creating the simplicial data structure (SDS) to extract contextual, topological and proximity relations as auxiliary data for subsequent generalization process of polygonal geometric features including buildings. Several generalization operators - exaggeration, collapse, amalgamation and simplification - employing auxiliary data in terms of neighbourhood relations on this structure for generalization have been described at length. In their approach, modification to conventional Delaunay triangulation was done in such a way that edges of geometric objects were enforced as constraints in the triangulation (Paul Chew, 1989; Domiter and Žalik, 2008).

Ware and Jones (1996) have also adopted constrained Delaunay triangulation (CDT) in creating SDS for detecting and resolving the conflict of polygonal building objects caused by scale reduction in the application of map generalization. They used a proximal search procedure with the use of vertices and edges of triangles of the CDT for solving nearest neighbouring polygonal building objects to identify and resolve conflicts between buildings. However, they have identified exceptions to the direct edge connectivity between nearby building objects thereby losing nearest neighbours, where a long constraining edge of a building in the CDT lies close to the vertices of the boundary of another building object (e.g. no neighbourhood relation between building identification number (IDN) 1 and 2 in Figure 2). Therefore, CDT can be considered as Delaunay unstable since all the triangles formed do not yield to the so-called empty circle criterion (Rognant *et al.*, 1999).

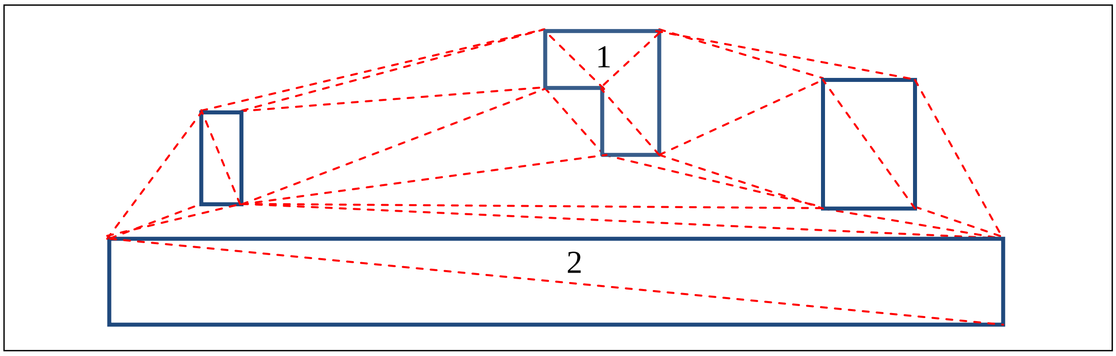


Figure 2: Constrained Delaunay triangulation of a set of building objects with constraining edges are shown in solid lines while other virtual edges are shown in dashed lines. The figure shows that there is no object link connectivity between building identification number (IDN) 1 and its nearest neighbour IDN 2.

Li *et al.* (2004) have used Delaunay triangulation to form initial building groups with the adjacency relations using every true connection triangle (Figure 3) that belongs to either two or three buildings in the triangulation by forming an adjacency matrix for buildings, recording the topological adjacency and minimum distance, together with other Gestalt factors such as free space, size, shape and orientation. By means of this matrix, further formation of groups consisting of more than two buildings is carried out with the help of graph theory assisted by the target scale and the Voronoi regions of the triangulation for the subsequent building generalization process using aggregation generalization operation. It can be seen from the results of the Delaunay triangulation shown in Figures 7 in their paper that their approach is based on constraint-breaking method as explained by Rognant *et al.* (1999), where each edge is densified - either each edge is split into sub-segments by adding artificial vertices or artificial vertices are interpolated on each edge of buildings without splitting edges into sub-segments to derive neighbourhood relationships between them. Ai *et al.* (2007) applied similar edge densification by splitting long boundary edges into sub-segments to have more triangle hooks between buildings to derive all possible neighbourhood relations.

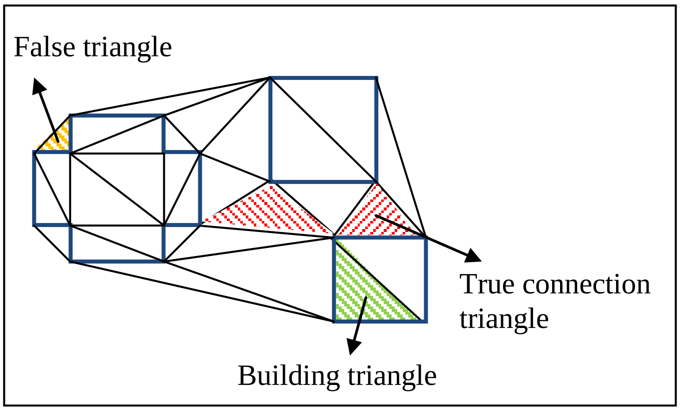


Figure 3: Default Delaunay triangulation considering vertices of all buildings as site points, based on Yan, Weibel and Yang (2008).

Regnauld (2003, 2005) and Regnauld and Revell (2007) have used the graph-based technique to cluster building polygons into groups based on triangulation. A graph is a list of nodes and edges connected to each other. Further, Regnauld (2005) introduced a data model based on object orientation concept to store any graph including triangulation with basic classes - graph, node and edge. According to this model, in order to store proximity and topology in the proximity graph using constrained Delaunay triangulation of building polygons, each polygon is stored as a node with X and Y coordinates and each line connecting two polygons as an edge after the construction of the triangulation. Also, the minimum distance between two connected building polygon pair is stored as a weight in the edge class (Figure 4(a)). In deducing the proximity graph with minimum distance as one of the attributes of the edge class from triangulation, when several edges of the triangulation connect two building polygons, triangles that connect these building polygons are retrieved and used to find such distance. Since constrained Delaunay triangulation has been used based on a 'divide-and-conquer' algorithm, it can create implicit proximity relations and may even lose important nearest neighbours as discussed by Ware and Jones (1996) especially when building objects have longer edges in the source data set.

Qi and Li (2008) have used a similar approach to cluster building groups using constrained Delaunay triangulation, proximity graph and finally minimum spanning tree, considering proximity, orientation and similarity used as weights in the tree. They have transformed adjacency relationship of building polygons to that of point objects to create the connectivity graph. Then with the links of each building pair in the graph, weights, proximity, orientation and similarity have been calculated. Since they have used the adjacency relationships created by constrained Delaunay triangulation, links in the connectivity graph may not represent the exact neighbourhood relations between building polygons owing to the implicit neighbourhood relations represented in constrained Delaunay triangulation as discussed above.

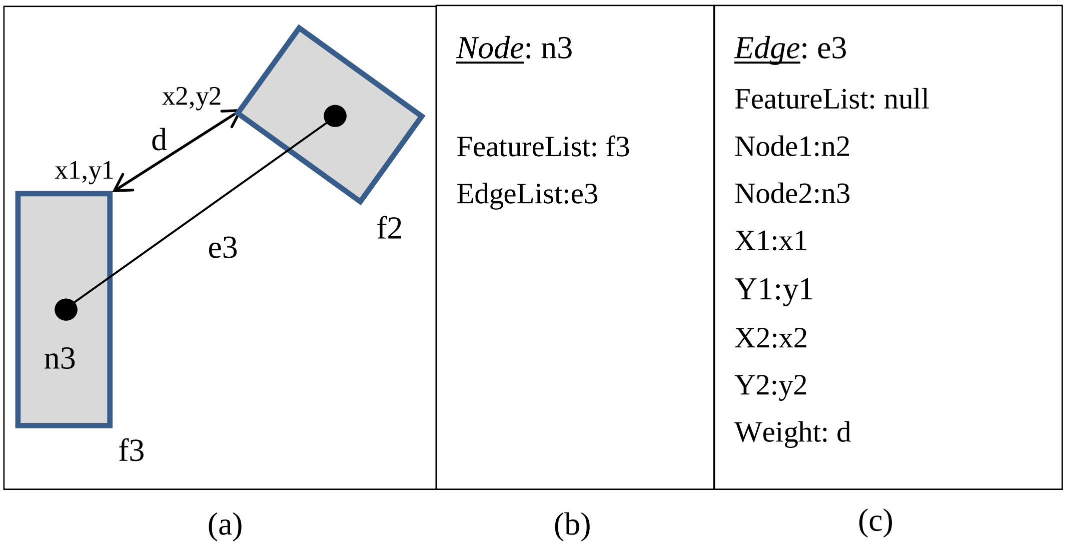


Figure 4: (a) Proximity graph between a building pair (b) Attributes of node: n3 and, (c) Attributes of edge: e3

Yan, Weibel and Yang (2008) have applied Delaunay triangulation to find the topological adjacency relations between buildings to be used for the calculation and recording of Gestalt factors such as minimum distance, area of visible scope of a building pair, size, shape, internal orientation and direction based on direction Voronoi diagram (DVD) required for building grouping process. For the detection of potential building pairs initially, they mentioned that they have adopted constrained Delaunay triangulation employed by Jones, Bundy and Ware (1995). When analysing their results, they have used Delaunay triangulation where each edge of the building is either split into sub-segments or artificial vertices are interpolated on each edge of a building without splitting it into sub-segments to derive neighbourhood relations. This approach is almost similar to those of Li *et al.*(2004) and Ai *et al.* (2007). However, triangles formed with the three nodes that belong to the same building (building triangles and false connection triangles in Figure 3 are omitted from the triangulation array since they are not used to detect adjacency relationship between two buildings).

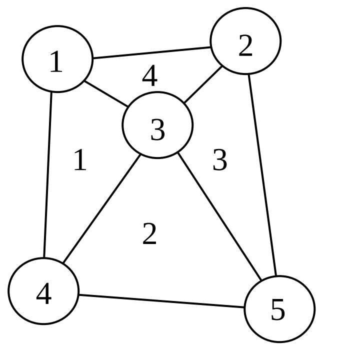
When reviewing the above work related to data enrichment on building features using Delaunay triangulation, mainly two triangulation techniques have been applied: constrained Delaunay triangulation and Delaunay triangulation with constraint edges based on the edge breaking principle. The first method creates implicit neighbourhood links between buildings since the triangulation does not yield the empty circumcircle criterion. The second method introduces new vertices that are not part of the original data set. In the second method, depending on the distance used to break the edges, there is no guarantee that all topological relations between buildings are formed. The next section will introduce a new approach based on Delaunay triangulation with edge constraints without adding additional vertices on edges to enrich spatial links, which is one of the horizontal relations of data enrichment of building features.

3. CONSTRUCTING CONDITIONAL DELAUNAY TRIANGULATION

Computation of Delaunay triangulation is based on the so-called recursive edge-flipping technique (Berg *et al.*, 2008) used to satisfy Delaunay's condition applied to triangles formed from a set of points. The algorithm of the approach presented in this paper for data enrichment using Delaunay triangulation is based on the incremental algorithm as mentioned in Section 1 because of its simplicity and flexibility, adding point by point to growing the existing triangulation.

3.1 Data structure

There are many possible data structures for representing triangulations. A data structure must be chosen in view of the requirements of the intended application. The data structure adopted in this approach is the minimal triangle-based data structure (Hjelle and Dæhlen, 2006) in order to have less storage and higher efficiency of carrying out topological and geometrical operations (Figure 5).



Triangle #	Vertex IDs		
1	1	4	3
2	3	4	5
3	2	3	5
4	1	3	2

Figure 5: (a) A simple triangulation consisting of building edges with vertex-ids 1 to 5, represented by building feature IDNs (b) minimal data structure of the simple triangulation with triangles and nodes of each triangle ordered counter-clockwise.

In adopting this data structure, each vertex of the point set ***P*** is assigned the feature IDN of the building polygon that the vertex belongs to, for the clear representation and maintenance of the topological and proximity relations between polygonal buildings.

3.2 Conditional Delaunay triangulation approach

The Delaunay triangulation algorithm used in this approach falls into the category of an incremental insertion algorithm as described in Section 1. The triangulation initially starts with a single triangle. Then the points are incrementally inserted one by one into the interior or the exterior to the initial triangulation. The duplicate points are ignored when adding new points to the triangulation. The sequence of steps of the constrained algorithm on Delaunay triangulation is described in the triangulation flow chart depicted in Figure 6.

Once the default Delaunay triangulation ($\mathbf{D}\Delta_n$) step is complete as depicted in Figure 6 on the set of site points which are the building vertices, the next step is to identify and delete triangle edges that run across polygon edges used as constraint edges (Shewchuk, 1999) and edges of building triangle as described in Figure 3 from triangles since no constraints were applied during default Delaunay triangulation.

To identify the edges of such triangles, 2D topological relations are used. Both intersection and cover relations (Egenhofer, Litwin and Schek, 1989) are checked between each triangle in the triangulation array against all building polygons in the data set. This is a time-consuming process when the data set is large and, therefore, spatial indexing should be used to improve the efficiency. If such a crossing triangle or a building triangle (Figure 7) is found, the triangle is removed from the triangulation array. This topological intersection/cover search criterion is checked for all triangles in the triangulation array finally to remove all crossing and building triangle edges.

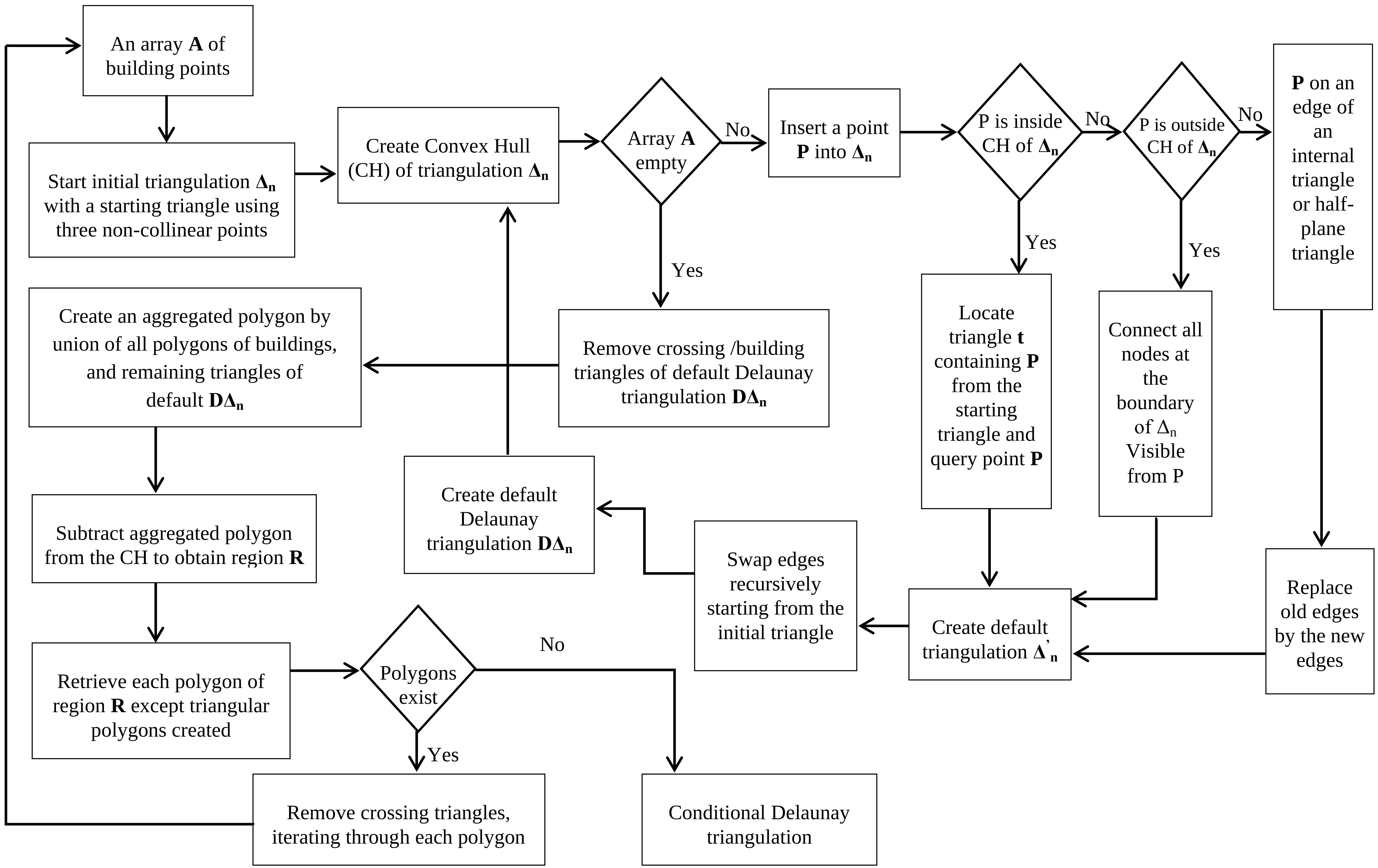


Figure 6: Conditional Delaunay triangulation approach based on Delaunay triangulation algorithm

The removal of both crossing triangles and building triangles from the final triangulation array is required to obtain valid proximity relations between building geometries. In this process, removal of crossing triangles leaves the triangulation invalid in the sense that every point falling in the space region ***R*** that is the area obtained by subtracting the summation of area of each building polygon from the convex hull of all site points ***P***, does not belong either to an edge or a vertex of one or more triangles or the interior of a single triangle. This instance is clear from the trapezoidal area created after the removal of crossing triangles from the triangulation in Figure 7.

Next step is to identify the polygons that need to be re-triangulated due to the removal of crossing triangles created in the initial Delaunay triangulation process as given in the triangulation approach in Figure 6. Triangles generated caused by the removal of crossing triangles after subtraction (see hatched triangle (middle) in Figure 7) are not re-triangulated and added to the triangle array to include missing triangles of the region ***R***. The other convex and/or concave polygons (Figure 7) with four sides or more are re-triangulated and added to the triangulation array iteratively until no polygon to be triangulated to complete the triangulation process. In this polygon re-triangulation process, crossing triangles formed as a result of concave polygons are required to be searched again and removed if found. Once the triangulation process is complete, the union of all triangles in the constrained triangulation must be equal to the region ***R*** over which the triangulation is defined, that is, R = U $\boldsymbol{t}_{i,j,k}$ where i, j and k refers to vertex IDNs of the triangle t.

It is important to note that the triangulation handles typical cases of site point configuration in the following manner: (a) no triangle is created if three points are collinear (b) duplicate points are ignored in the triangulation and (c) if the four points of a convex quadrilateral are co-circular, the choice of the triangulation depends on the point order of each triangle.

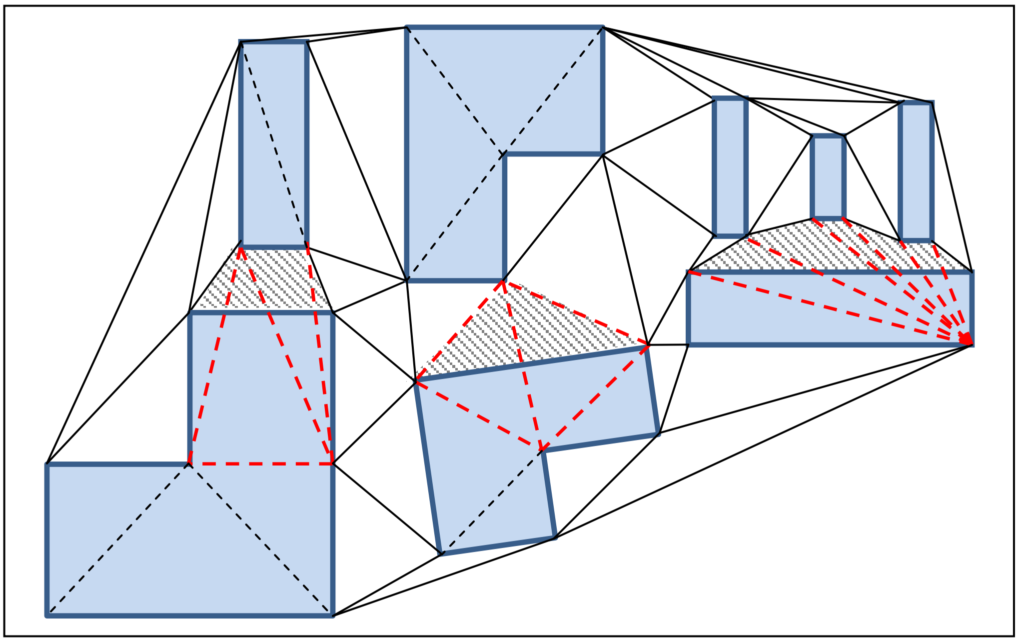


Figure 7: Crossing triangles to be removed in the triangulation process are shown in broken red lines. Remaining isolated polygons hatched including a convex polygon (left), a triangle (middle) and a concave polygon (right) after the removal of initial crossing triangles.

4. IMPLEMENTATION

The Conditional Delaunay triangulation (CnDT) approach described in the previous sections has been implemented as a prototype in Java object-oriented programming language with a Graphical User Interface (GUI). Geographic features are stored in open-source PostgreSQL database with PostGIS extension (PostGIS Home, 2014) to handle spatial data. The GUI being the front end has the capability to talk to the database at the backend to process data. The advantage of this setup is that not only a large volume of data can be processed but also the ability to work in the client-server environment if the database can be hosted.

The tests to generate CnDT and its neighbourhood relations with proximity have been performed on a Laptop with an Intel Core 2 Duo 2.5 GHz processor, and with 4GB of RAM (3.5GB usable). The Default Delaunay triangulation took 110ms while CnDT took 1200ms on a data set consisting 125 building features. Table 1 shows the execution times required to build triangulation and neighbourhood relations over a large set of features.

Geometric entities	**Triangulation**		**Neighbourhood relations**	
Features/Nodes	*Default Delaunay triangulation (ms)*	*Conditional Delaunay triangulation (ms)*	*Number of Proximity links*	*Execution time of Proximity links (ms)*
125/1015	110	1200	364	560
250/2156	155	2370	751	1750
500/3522	170	3460	1504	3800
1000/5879	245	5200	3052	9140

Table 1: Computation times to generate triangulation and neighbourhood relations

5. RESULTS

Figure 8 illustrates the capabilities of the prototype developed. The new conditional Delaunay triangulation (CnDT) has been computed on both simple and complex building polygonal geometries together with proximity derivation of neighbourhood buildings. The complex geometry includes irregularly spaced building vertices on building edges, contiguous buildings, and buildings with holes in the raw data set (Figure 8(d)). The same approach can be used for the computation of CnDT and for deriving rich neighbourhood relations between any type of polygonal features and not restricted to building polygons.

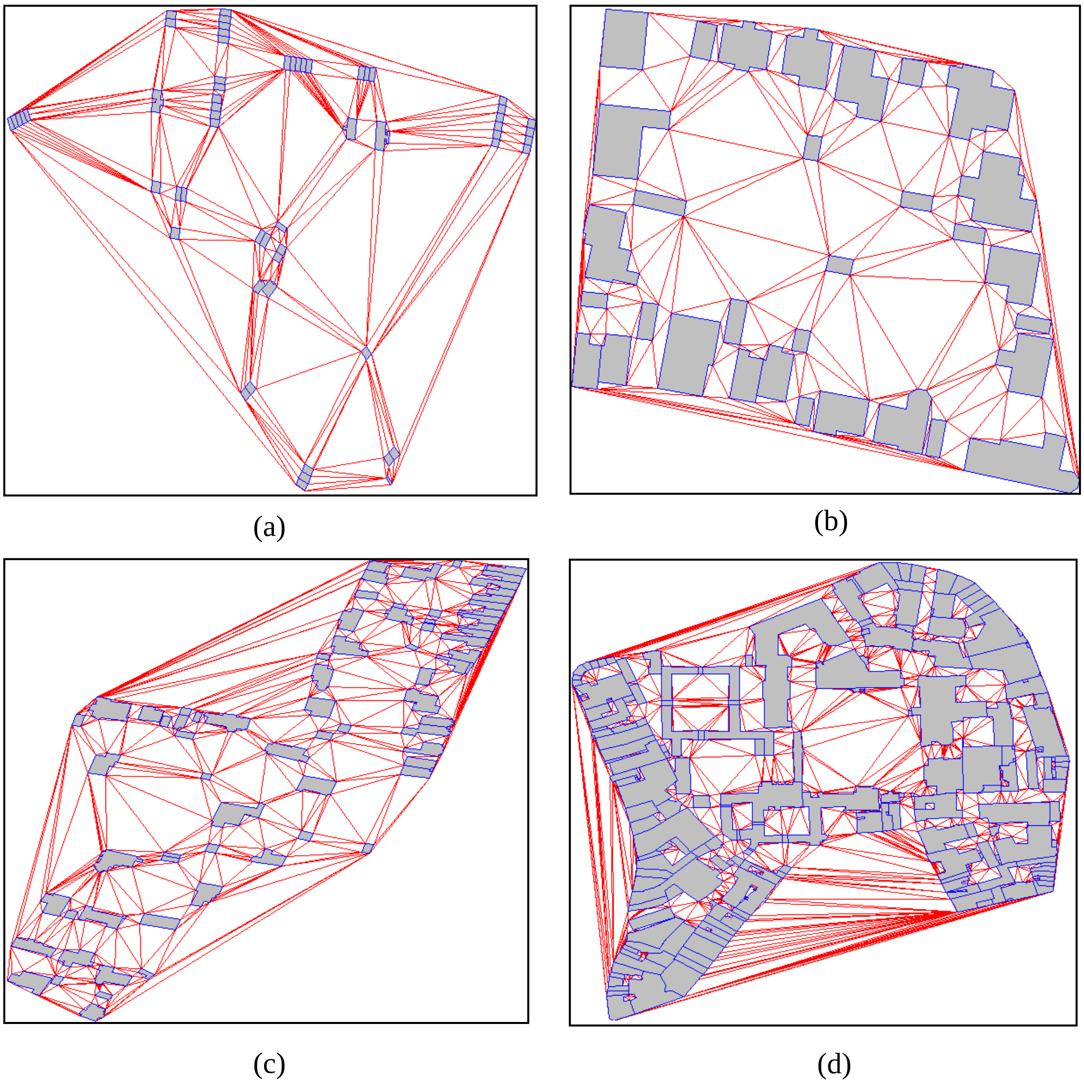
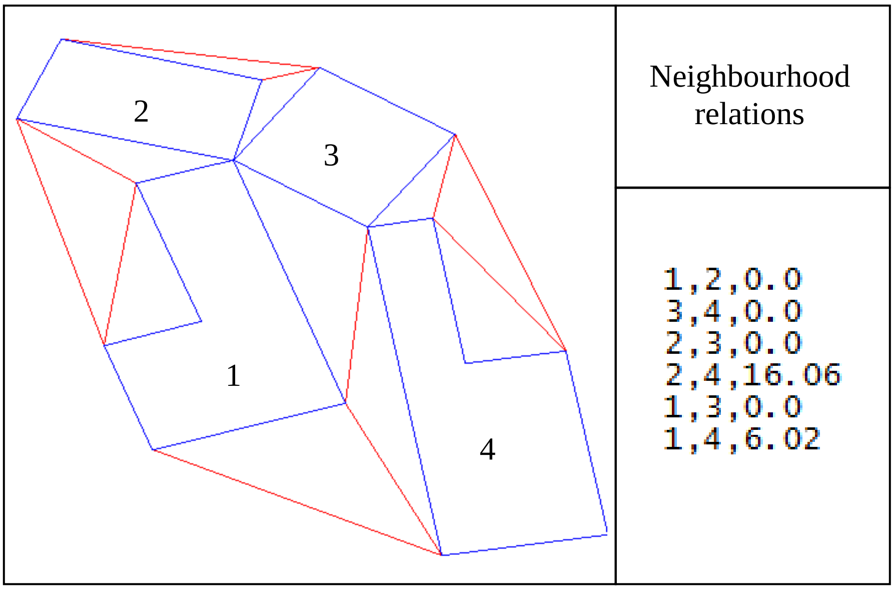


Figure 8: Conditional Delaunay triangulation. Examples from (a) to (d) represent data sets from simple building geometry to very complex building geometry, consisting of rectangles to differently shaped buildings including attached building edges and holes, located at different distances. Source: Ordnance Survey MasterMap data at the scale of 1 : 1.25K, Crown copyright.

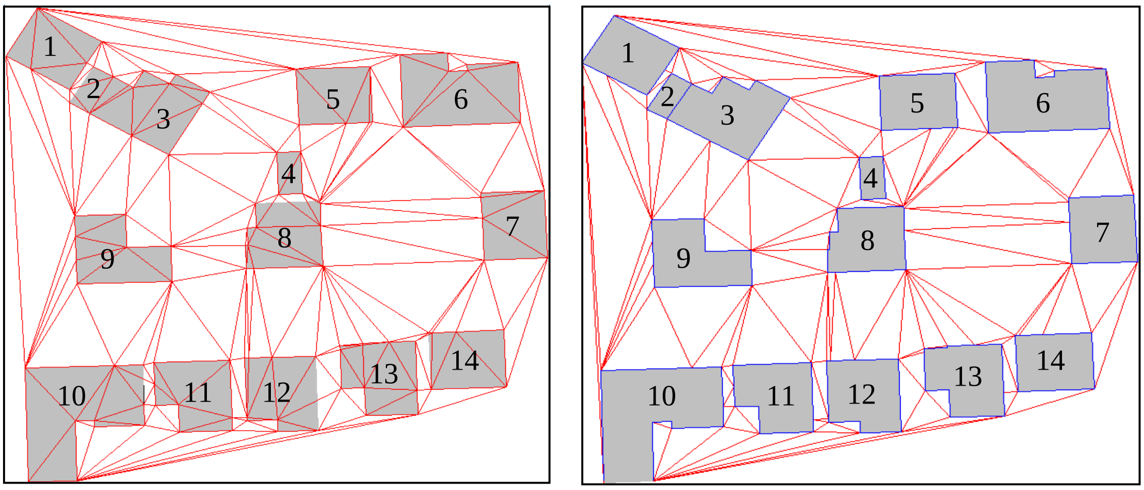
Figure 9 clearly illustrates the neighbourhood and proximity relations derived from a conditional Delaunay triangulation on a synthetic data set for the purpose of easy clarification.



(a) (b)

Figure 9: Conditional Delaunay triangulation formed by a simple synthetic data set: (a) Buildings with identification numbers surrounded by Delaunay triangles, (b) Neighbourhood relations with proximity in terms of minimum distance between buildings in the form of [*Builidng_ID_From*, *Building_ID_To*, *Minimum_Distance*] in ASCii format.

Figure 10 illustrates the results of Default Delaunay triangulation and Conditional Delaunay triangulation (CnDT) applied to a set of buildings. When comparing Figures 10(a) and 10(b), it is clearly observed that CnDT has respected the edge constraints between buildings 1 and 2, 4 and 8, and 10 and 11 while Default Delaunay triangulation does not respect the edge constraints.

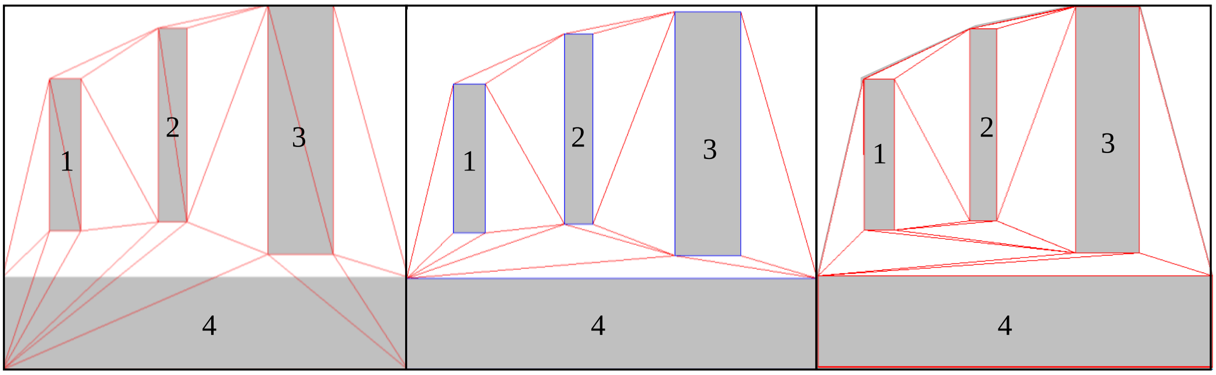


(a) (b)

Figure 10: Comparison between Default Delaunay triangulation and conditional Delaunay triangulation. Source: Ordnance Survey MasterMap data at the scale of 1 : 1.25K, Crown copyright.

A comparison of proximity relations and neighbourhood relations generated both on a synthetic data set and real data sets separately using the new conditional Delaunay triangulation (CnDT) algorithm and the Constrained Delaunay triangulation algorithm (CDT) based on the sweep-line algorithm developed by Domiter and Žalik (2008), implemented using the open source Poly2Tri CDT library (poly2Tri, 2014) in this work is illustrated in Figures 11 and 12. In this sweep-line algorithm, all points are sorted with reference to Y coordinate and the starting triangle with the advance front segment being two of its edges is created after sorting and analysing the vertices. The sweep-line is moved along the Y-axis and stops at every point it meets. Then by determination of the location of projection of each such point on the advance front segment, new Delaunay triangles are created with an updated advance front segment. Once triangles thus created cross a constraint edge, intersecting triangles are removed and empty areas are triangulated with default triangulation without swapping triangle edges served as constraint edges, generating triangles with weaker Delaunay property as depicted in Figures 11(c), 12(c) and 12(f).

According to the results of synthetic data in Figure 11(a), default Delaunay triangulation does not respect constraint edges of buildings. It can be clearly understood from the results of CDT in Figure 11(c) that the link between nearest neighbouring buildings 2 and 4 in the synthetic data set is lost causing implicit proximity links, while neighbourhood relations and explicit proximity links are represented in the new CnDT algorithm (Figure 11(b)).



(a) (b) (c)

Figure 11: (a) Default Delaunay triangulation, (b) conditional Delaunay triangulation, and (c) constrained Delaunay triangulation, generated on a synthetic data set. Source: Ordnance Survey MasterMap data at the scale of 1 : 1.25K, Crown copyright.

Some of the results obtained using the default Delaunay triangulation on two different real data sets with places labelled with ‘A’, ‘B’, ‘C’ and ‘D’ in Figures 12(a) and 12(d) do not respect the building edge constraints. When comparing the results obtained using the new CnDT algorithm developed in this work with those obtained using the CDT algorithm by Domiter and Žalik (2008) as depicted in Figures 12(b) and 12(c), and 12(e) and 12(f) respectively, results of adjacency relations between building IDNs 1 and 2, 2 and 5, and 4 and 6 generated using CDT are implicit in Figure 12(c) and the results of adjacency relations of building IDNs 1, 2 and 3 generated using the same CDT are implicit as depicted in Figure 12(f) when compared to those obtained using CnDT as shown in Figures 12(b) and 12(e). These non-Delaunay compliant triangles and implicit neighbourhood relations are generated because of the non-swapping of edges of triangles to be able to serve as constraint edges in the CDT algorithm as described above. Such non-Delaunay compliant triangles with constraint edges would satisfy the modified circumcircle criterion of CDT as explained in Section 1 above.

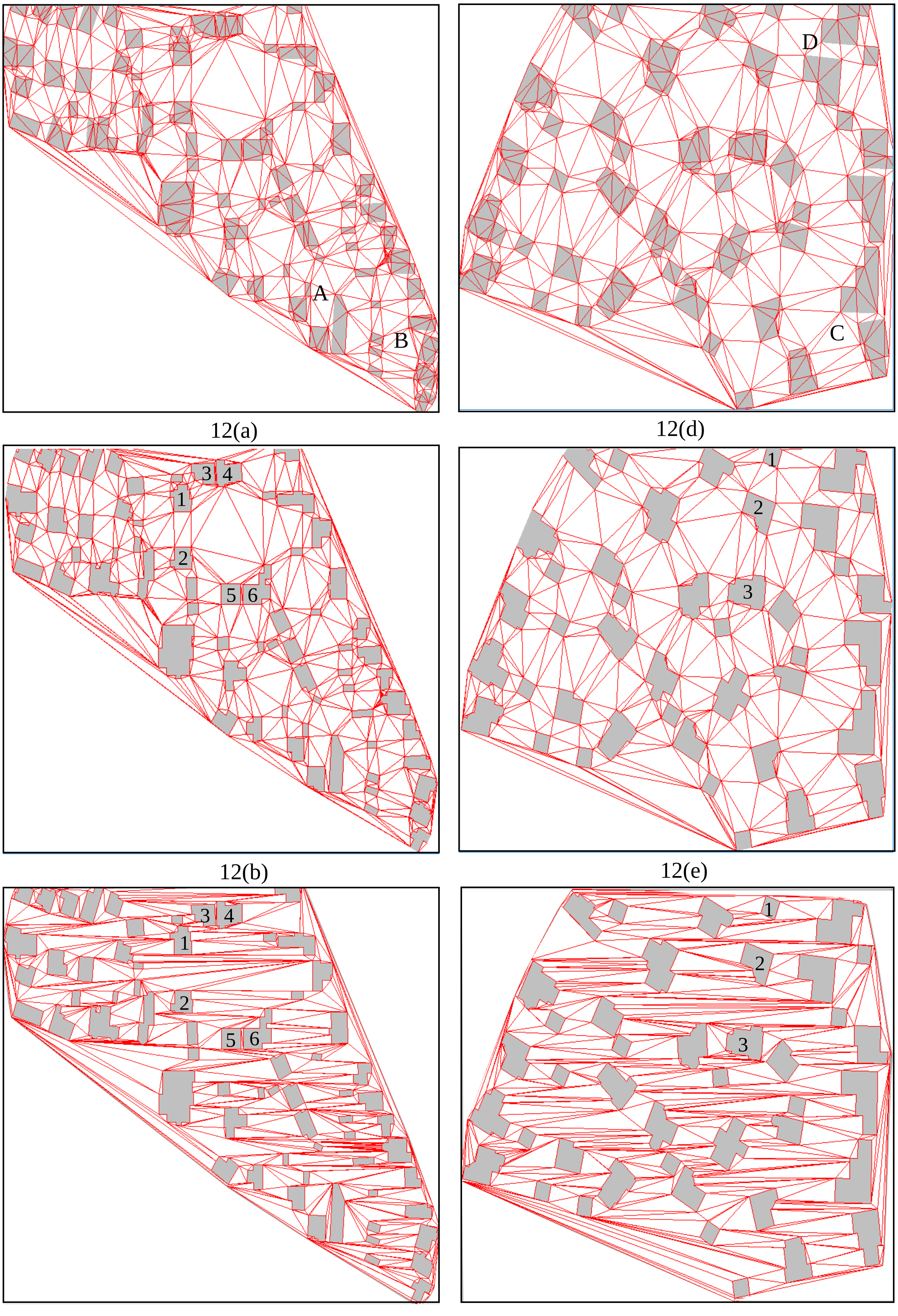


Figure 12: Results of Default Delaunay triangulation: (a) and (d), conditional Delaunay triangulation: (b) and (e), and constrained Delaunay triangulation: (c) and (f), generated on two different data sets. Source: Ordnance Survey MasterMap data at the scale of 1 : 1.25K, Crown copyright.

6. DISCUSSION

When observing the results in Figures 10, 11 and 12 of the new conditional Delaunay triangulation (CnDT) developed in this work, it can be understood that CnDT provides explicit neighbourhood relations, enabling the extraction of rich proximity relations between building polygons from very simple geometry to complex geometry including buildings with shared edges, holes and edges irregular splits without adding any new vertices during triangulation process (i.e., source data is not affected and kept intact). Since the new CnDT algorithm is entirely based on Delaunay triangulation, not constraining building edges during triangulation, the triangulation is Delaunay stable and its triangles have the property of being as equilateral as possible (i.e., empty-circumcircle criterion is respected in which the circumcircle going through any three site points in the triangulation must not have any other site points inside), avoiding the creation of skinny triangles. However, due to re-application of Delaunay triangulation on polygons created after removing crossing triangles as described in Figure 6 above, there can be a few triangles that do not respect this criterion. Nevertheless, this does not affect generating explicit neighbourhood relations. One of the examples of this situation is depicted in Figure 11(b) where the circumcircle of the triangle immediately on top of building IDN 4 covers two bottom corners of building IDN 4. One of the limitations of this new CnDT algorithm is that it can handle only polygons in generating triangulation, and a combination of line geometries and polygon geometries (e.g., roads and buildings) cannot be processed in the triangulation.

In comparison to new CnDT in this work, a significant drawback of constrained Delaunay triangulation (CDT) on edge forcing principle though used by Jones, Bundy and Ware (1995) and Ware and Jones (1996) for enriching spatial relationships between building polygons for subsequent automated map generalization, is that they do not provide explicit neighbourhood relationships between polygon geometries as nearby vertices would not be connected if there is an intervening edge which is a boundary of a building. This constraint and the existence of building edges with several irregularly spaced splits in source data leads to creating skinny triangles that do not respect empty-circumcircle criterion, creating implicit neighbourhood relations as evident from the results in Figure 12(c) and 12(f).

The Delaunay triangulation approaches by Li *et al.* (2004), Ai *et al.* (2007) and Yan, Weibel and Yang (2008) are based on introducing artificial vertices on building edges (densification of edges based on edge breaking principle) to derive explicit neighbourhood relations on any type of data set. For the purpose of deriving such vertices, a proper interpolation interval threshold will have to be determined and used, and it will vary from data set to data set depending on the nature of data (some edges in the source data may have irregular splits). In this process, if sub-segments are created by splitting existing edges, it affects source building data, necessitating the removal of any artificial vertices added if building edges are to be simplified in subsequent automated map generalization. If artificial vertices are interpolated and used to form artificial sub-segments without splitting edges of buildings, such sub-segments become edges of triangles in the triangulation. If triangles in such a triangulation are used to bridge gaps between buildings in creating building amalgams in subsequent automated map generalization, topological exceptions may occur unless such interpolated sub-segments are not exactly on the original building edges in the source data. Since no densification of edges is performed in the new CnDT approach, original source data is not affected. Hence, no additional processing of data is especially required to avoid exceptions in automated map generalization applications as mentioned above.

7. CONCLUSION

This paper proposes a new approach to derive explicit topological and proximity relations between building polygonal features with complex geometries using Delaunay triangulation data structure. Although there is some functionality available based on Delaunay triangulation in existing proprietary and open-source GIS software, they do not provide the flexibility and richness of tools required to extract data required by developers for automated map generalization. The term conditional Delaunay triangulation (CnDT) is assigned because all the triangles formed have the condition of being as equilateral as possible, and the each edge of every building is an edge of the Delaunay triangles thus formed. To apply the edge constraint, a new principle termed as 'non-edge intersection' is adopted other than the two principles used in other triangulation methods - 'edge forcing' and 'edge breaking' - discussed in the literature for subsequent automated map generalization. The representation of rich and explicit neighbourhood relations between polygonal buildings also enables deriving rich proximity relations from the building pairs connected by true connecting triangles. The enrichment of spatial relations using CnDT not only supports the derivation of Gestalt factors such as minimum distance, orientation and similarity of spatial objects but also to formulate generalization operations such as object amalgamation, collapsing, exaggeration and displacement based on various algorithms in the field of automated map generalization. The algorithm has been implemented in Java using open-source class libraries and runs fully automatically with a Graphical User Interface (GUI). The results of the triangulation emphasise that the triangulation can handle building polygons with contiguous edges, holes and irregularly placed vertices on edges of buildings. However, as described in the previous Section, one of the limitations of this algorithm is that it can only be used with polygon geometries. Further work is necessary to improve this algorithm to handle both linear and polygon geometries in deriving spatial relationships between objects.

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