Establishing a Horizontal Velocity Model of Taiwan Using GPS Observations and the Least-squares Collocation Technique

Ching-Jung Hung[1] and Ming Yang[2]
[1]Graduate Student, Department of Geomatics, National Cheng Kung University
Tainan 70101, Taiwan; phone: +886-6-2370876 ext. 832;
Email: P66984100@gmail.com

[2]Professor and chairman, Department of Geomatics, National Cheng Kung University,
Tainan 70101, Taiwan; phone: +886-6-2757575 ext. 63820;
E-mail: myang@mail.ncku.edu.tw

KEY WORDS: International Terrestrial Reference Frame, Geodetic Control, Horizontal Velocity Model, Least-squares Collocation

ABSTRACT: Taiwan is located at the boundary between the Eurasia plate and Philippines sea plate, where active plate motion leads to significant annual displacements of geodetic control points up to 4-5 cm. For the purpose of maintaining the existing Taiwan Geodetic Datum 1997 (TWD97) horizontal coordinate system which is connected to the International Terrestrial Reference Frame (ITRF), this research aims at predicting the yearly displacements of thousands of control points of various orders by establishing a horizontal velocity model of Taiwan. This velocity model is composed of two groups of physical parameters which characterize the regional block motions and the residual systematic signals, respectively. We used GPS-derived velocity observations collected at 472 points from 1993 to 2005 and the Least-squares Collocation (LSC) technique to create this model. And for validating the model, we further adopted 16 evenly distribute continuous GPS tracking stations as external check points. In stable regions, the accuracy of validation is ±5.633 mm/year. The results show that the developed model is capable of providing accurate estimates of annual displacements of geodetic control points in Taiwan.

1. Introduction

The existing Taiwan geodetic datum 1997 (TWD97) datum is connected to the International Terrestrial Reference Frame (ITRF) and established by the GPS positioning technique in 1997. The GPS fundamental network in this datum is composed of eight permanent tracking stations, 105 first-order and 621 second-order GPS stations (Yang et al., 2001). Since survey engineering, civil engineering base on geodetic control points, the quality of geodetic control points play an important role in development of Nation and the right of people.

However, Taiwan is situated on the tectonic convergence boundary of the Eurasian and Philippine Sea plates, moving at a rate of several centimeters per year (DeMets et al., 1990; Yu et al., 1997). Hence, the frequent and severe motion of two plates must lead to the datum continually increasing distortion. It is inevitable that the geodetic control points would be changed by active plate motion.

Global Positioning System (GPS) is not restricted to weather, region and provided with high-accuracy positioning; also, GPS has been generally applied in monitoring plate motion and studying active tectonics (Dixon, 1991; Kotzev et al., 2001).

To acquire the displacement of the coordinate of geodetic control point, we can calculate angle velocity of XYZ-axis for each plate to build up a plate motion model by Euler theorem. Legrand *et al.* (2006) adopted the horizontal velocity fields at 78 GPS stations from 1996 to 2006 to establish the velocity model in Western Europe by Euler theorem; however, experimental result showed a systematic error exists in the estimated velocity field.

The technique of Least-squares collocation (LSC) has been developed from the study of interpolating global gravity anomaly (Moritz, 1978). It is a useful and powerful method to apply for variety problems in interpolation and prediction. Since the method not only deal with random error but systemic error at arbitrary points. In recently years, several researchers have utilized the Least-squares Collocation to interpolate the velocity field of geodetic point (Kato et al., 1999; Wu et al., 2006; Hefty, 2007).

Therefore, in this research, in order to approach the velocity of geodetic point, we establish a horizontal velocity model of Taiwan using GPS observations and the least-squares collocation technique.

2. The Block Motion Theorem

2.1 Euler parameters

In order to describe the velocity which was influenced by the plate motion of geodetic control point, first we need to understand how the plates move. According to Euler theorem, any rigid block rotate particular angle to new location around an axis which pass through earth mass center, the axis called Euler rotation axis. Euler rotation axis intersection with the ground is rotating pole, also known as Euler pole (figure 2). Therefore, we learn the block motion by Euler pole and the angle velocity around Euler rotation axis.

Both of the ITRF and Euler rotation axis's origin are defined in the earth mass center, hence, it is directly and convenient to establish block motion model by Euler theorem and GPS observation based on ITRF. The mathematic equation of Euler theorem is expressed as follow:

$$V_i = r_i \cdot \omega \tag{1}$$

where V_i is the velocity of certain point *i* in block; *r* is the radius vector of certain point *i* in block; ω is the angular velocity around Euler rotation axis also called Euler parameter

Furthermore, the velocity component in north-south V_n and east-west V_e directions of the site with coordinates φ and λ are expressed as:

$$\begin{bmatrix} V_e \\ V_n \end{bmatrix} = \begin{bmatrix} -R\cos\lambda\sin\varphi & -R\sin\lambda\sin\varphi & R\cos\varphi \\ R\sin\lambda & -R\cos\lambda & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

where *R* = earth radius approximate 6370km

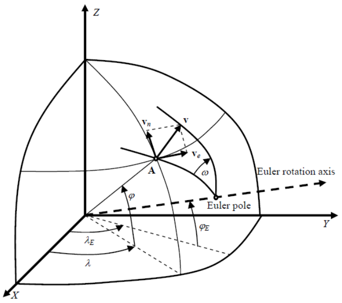

Figure2. Euler theorem diagram. (Hefty, 2007)

2.2 Separating Blocks

Because Taiwan is located in the area of active plate motion, in order to indentify the motion in each part of Taiwan reasonably, this study adopt the strategy of Ching *et al.* (2011) , regarding the faults as borders, and divided Taiwan into 27 blocks which are EURA, YMSD, WNOR, TAOC, CHMO, MIAO, WTFH, SHSS, WCPN, CHYI, WCPS, EKSH, LUCU, KHSC, PINT, EFSB, SCEN, PHSP, LVFD, CENT, NCOR, NCEN, RYUK, ILAN, NHSS, NORT. Therefore, through Euler theorem and the divisions of blocks, we can sufficiently show the displacement of geodetic control point in Taiwan.

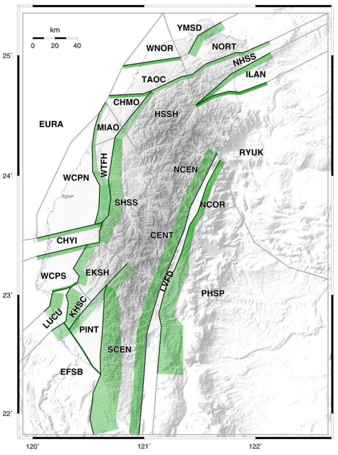

Figure3. The location of each blocks (ching et al., 2011)

3. The Method

3.1 Least-squares Collocation

The basic equation of least-squares collocation is expressed as follow (Moritz, 1989)

$$l = Ax + Bs + n = Ax + B\begin{bmatrix} t \\ u \end{bmatrix} + n \tag{3}$$

where l is vector of observations; A *is* coefficient matrix; x is vector of parameters; B *is* matrix which is $[I \quad 0]$; I is unit matrix; s is signal vector which contain signal vector t at observation points and the signal vector u at the non-observation points; n is noise vector.

In Equation 3, the parametric part Ax denotes velocity of each geodetic point in the block was determined by block motion. Here, the vector x indicates that the angular velocity of each block. The signal s represents that velocity was determined by the slip rate of fault and the like.

The solutions of parameter $\hat{x}$, signal $\hat{s}$ and noise $\hat{n}$ are listed as follows (Moritz, 1989)

$$\begin{cases} \hat{x} = (A^T(C_{tt}+C_{nn})^{-1}A)^{-1}A^T(C_{tt}+C_{nn})^{-1}l \\ \hat{s} = \begin{bmatrix} \hat{t} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} C_{tt} \\ C_{tu} \end{bmatrix}(C_{tt}+C_{nn})^{-1}(l - A\hat{x}) \\ \hat{n} = C_{nn}(C_{tt}+C_{nn})^{-1}(l - A\hat{x}) \end{cases} \tag{4}$$

where C_{tt}, C_{tu}, and C_{uu} are covariance matrices

Mikhail and Ackerman (1976) suggested an equation to estimate covariance functions under the assumption of homogenous and isotropic. The equations of variance and covariance can be expressed as follows:

Variance: $$C_{\hat{l}}(0) = \frac{1}{N}\sum_{i=1}^{N}\hat{l}_n^2 \tag{5}$$

Covariance: $$C_{\hat{l}}(d_p) = \frac{1}{N_p}\sum_{i<j}^{N_p}\hat{l}_i\hat{l}_j \tag{6}$$

Where $\hat{l}$ is parametric part Ax is removed from observations l, N is numbers of data points, N_p is numbers of data points with a specific distance interval d_p.

Since the trend of variance and covariance function in this study is consistent with the exponential function, we select the exponential function to fit it. The exponential function can be expressed as follow:

$$C_s(d) = C_s(0)\cdot\exp(-kd) \tag{7}$$

where $C_s(0)$, k are constant which derived by regression, d is the distance between data points.

3.2 Least-squares Collocation

To illustrate the experimental processes more clearly, we divide it into three steps (figure 4). First, we establish the velocity model which is composed of two groups of parameters with the Euler parameters and the covariance function by GPS velocity and least-squares collocation.

Second is parameter estimation. In this model, we just input the longitude, latitude and location block of arbitrary points; then the model calculate regional block motion and systemic signals respectively; finally, we get the velocity estimated by this model.

Third is validating the accuracy of this model. We tested the accuracy by comparing two kind of GPS-derived data sets. One data set was the GPS observations were used as internal check and developed velocity model, another data set estimated by GPS lab was used in external check and validated this model. (The velocity data are available at: http://gps.earth.sinica.edu.tw).

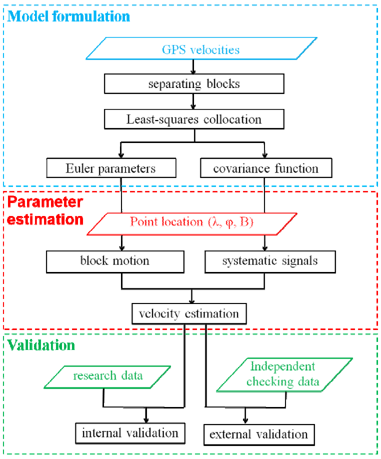

Figure4. The overall experimental flow chart.

4. Experimental Data

The GPS data in this study is obtained from Ching et al.(2011). In 1997 M_w7.6 Chi-Chi earthquake, the central Taiwan was the most damaged area. To avoid the co-seismic and post-seismic deformation which caused by Chi-Chi earthquake influencing the velocity of site in central Taiwan, we only use the GPS data for central Taiwan prior to the earthquake between 1995 and 1999 (Table 1). The distributions and vectors of GPS sites show in (Figure 5)

Period (Organization)	Numbers of data points	Location
1995-1999 (IES)	79	All over Taiwan (before Chi-Chi earthquake)
1996-1999 (CGS)	90	In central Taiwan (before Chi-Chi earthquake)
1995-2005 (MOI)	303	In north, south, east of Taiwan

Table1. The period and numbers of stations of GPS data

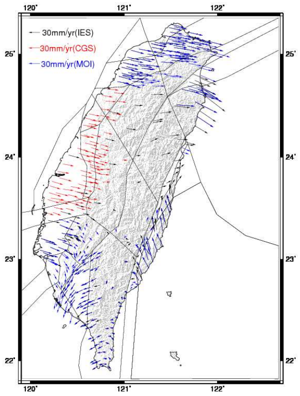

Figure5. The distributions and vectors of GPS sites.

5. Results and Discussions

5.1 Internal Validation

Here it is simply to validate the accuracy of the velocity model by internal validation. We adopted the GPS data in Table 1 as the check points. The RMS (see Table 2) were ±4.010 mm/year in the W-E direction and ±3.439 mm/year in the N-S direction. The histograms of RMS are shown in Figure 6.

Direction	Mean (mm/yr)	STD (mm/yr)	RMS (mm/yr)
W-E	-0.041	±4.010	±4.010
N-S	0.148	±3.436	±3.439

Table2. The values of mean, STD and RMS in the W-E and N-S direction.

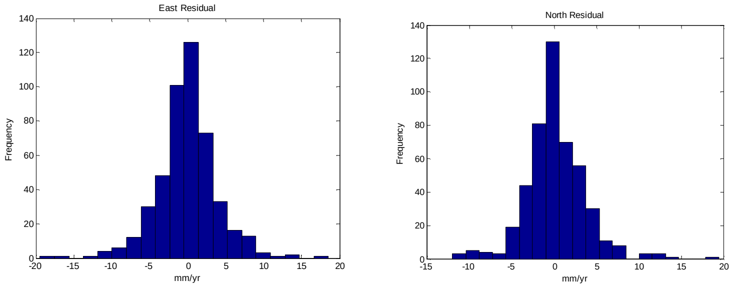

Figure6. Histograms of W-E (left) and N-S (right) direction residuals

We combined the residuals of the W-E and N-S direction and divided their magnitude into three ranges (0 to 3mm/year, 3 to 6mm/year and greater than 6mm/year). The three ranges are expressed as different colors and shapes respectively (see Figure 7a). The worst fit regions locate at the fault side. Figure 7b shows the slip rate of faults derived by Ching *et al.* (2011). By the comparison of the two figures, the regions with the residuals more than 6mm/year are consistent in the regions with the fault slip rate greater than 20mm/year, and also the region distributed over central mountains in Taiwan. The result indicates that the usage of this velocity model is restricted in the faults slip rate more than 20mm/year.

5.2 External Validation

The second data set used to validate this velocity model was estimated by GPS lab. We selected 16 GPS stations as evenly distributed check points. Figure 8a shows the name, location, GPS velocity derived by GPS lab (red vector) and GPS velocity estimated by this model (blue vector) of each station. Figure 8b indicates that residuals of external check points.

In internal validation, we know that the velocity model get worse result in fault slip rate greater than 20mm/year regions. Therefore, to validate the influence of fault slip rate, we separate check points into two groups which locate fault slip rate greater than 20mm/year and less than 20mm/year regions. Table 3 shows the result, the RMS of group1 (fault less than 20mm/year) are ±5.633mm/year, the RMS of group2 (fault greater than 20mm/year) are ±24.771mm/year, the RMS of overall check points are ±17.525mm/year.

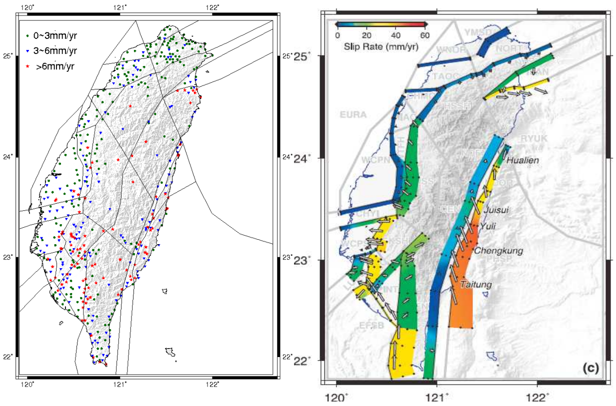

Figure7. **(a)** The residuals and distributions of inversion data **(b)** The fault slip rate (Ching et al., 2011)

Data set	Numbers of stations	Mean (mm/yr)	RMS (mm/yr)
Group 1 (fault slip rate <20mm/yr)	8	4.947	±5.633
Group 2 (fault slip rate >20mm/yr)	8	23.287	±24.771
Group 1+2	16	13.254	±17.525

Table3. The accuracy of external validation check points

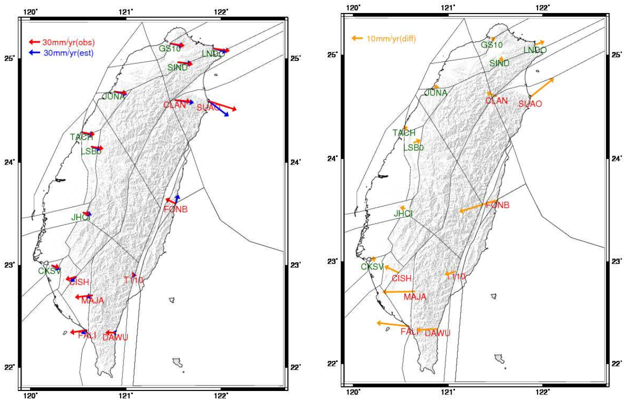

Figure8. (a) The velocity estimated by using this model (blue vector) and velocity derived by GPS lab (red vector); **(b)** The residuals of external check points.

6. Summary

In this research, we successfully establish a horizontal velocity model by using 472 GPS velocity and the technique of least-squares collocation. External validation of the velocity model shows, in fault slip rate less than 20mm/year regions, the RMS of this model reaches ±5mm/yr, indicating the accuracy of this model is well; nevertheless, in fault slip rate greater than 20mm/year regions, the RMS reaches about ±25mm/yr, is the restriction of this model. In the future, maybe we can overcome the restriction of this model by importing more and more GPS velocity.

References

Ching K-E, Rau R-J, Johnson KM, Lee J-C, HU J-C (2011) Present-day Kinematics of active mountain building in Taiwan from GPS observations during 1995-2005. Journal of Geophysical Research

DeMets C, Gordon RG, Argus DF, Stein S (1990) Current Plate Motions. Geophysical Journal International 101 (2):425-478

Dixon TH (1991) An Introduction to the Global Positioning System and Some Geological Applications. Reviews of Geophysics 29 (2):249-276

Hefty J (2007) Geo-Kinematics of Central and South-East Europe Resulting from Combination of Various Regional GPS Velocity Field. Acta Geodyn Geomater 4 (4):173-189

Juliette L, Zuheir A, Olivier J (2006) Interpolation of the European Velocity Field Using Least-squares Collocation Method.

Kato T, El-Fiky GS (1999) Continuous Distribution of the Horizontal Strain in the Tohoku District, Japan, Predicted by Least-squares Collocation. Geodynamics 27:213-236

Kotzev V, Nakov R, Burchfiel BC, King R, Reilinger R (2001) GPS Study of Active Tectonics in Bulgaria:Results from 1996 to 1998. Journal of Geodynamics 31:189-200

Mikhail M, Ackermann F (1976) Observations and Squares. Harper and Row, New York

Moritz H (1989) Advanced Physical Geodesy. Second Edition edn. Herbert Wichmann, Karlsruhe

Wu JC, Tang HW, Chen YQ, Li YX (2006) The Current Strain Distribution in the North, China Basin of Eastern China by Least-squares Collocation. Journal of Geodynamics 41 (5):462-470

Yu S-B, Chen H-Y, Kuo L-C (1997) Velocity Field of GPS Stations in the Taiwan Area. Tectonophysics 274:41-59

Yang M, Tseng C-L, Yu J-Y (2001) Establishment and Maintenance of Taiwan Geodetic Datum 1997. Journal of Surveying Engineering 127 (4):119-132