IMAGE RECONSTRUCTION ALGORITHM (STUDY) OF NON-EQUAL SPACING CMOS SENSOR OUTPUT RESPONSE

Cynthia Liu [1] , Nai-Yu Chen[2] and Jer Ling [3]

[1] Associate researcher, System Engineering Division, National Space Organization,
Hsin-Chu, Taiwan, 30078, Tel: 886-3-5784208 #1186;
E-mail: cynthia@nspo.narl.org.tw

[2] Research Fellow, Image Processing Division, National Space Organization,
Hsin-Chu, Taiwan, 30078, Tel: 886-3-5784208 #1331;
E-mail: nychen@nspo.narl.org.tw

[3] Research Fellow, System Engineering Division, National Space Organization,
Hsin-Chu, Taiwan, 30078, Tel: 886-3-5784208 #8441;
E-mail:jl@nspo.narl.org.tw

KEY WORDS: CMOS Image Sensors, non-mirror-type pixel layout, Time Delay Integration (TDI), Image Reconstruction.

ABSTRACT: With the self-developing CMOS imaging sensors in the instrument Focal Plane Assembly (FPA), there is flexibility to trade-off for optimal performance of CMOS sensors through systematic studies. The criteria considered for the optimization are MTF and SNR, the CMOS imaging sensor considered in this work is with TDI (time delay integration) feature.

Due to TDI sensor circuit complexity, fill factor (FF) will be decreased, the SNR is affected consequentially. Considering different fill factors, mirror-type and non-mirror-type pixel layouts are studied. Mirror type pixel layout may cause different responses between even and odd pixels. Analysis results based on the construction of the non-equal spacing signal via Whittaker-Shannon interpolation formula (Liu, 2010) show that the impacts of non-equal spacing in image signal are non-negligible. To avoid the impact on even and odd pixel output, change the layout to make up a symmetrical is one way, but will reduce the FF, Another way is to run pre-processing on ground before normal image processing chain.

It is possible to reconstruct a band-limited signal with non-equal spacing sampled data. KOHLENBERG 's paper published in 1953[3] shows that a function lies in a frequency band *(Wo, Wo+ W)* is completely determined by its values at a properly chosen set of points of density *2W.* The convolution kernel is derived for the spectrum of a multiply-periodic, amplitude modulated sequence of pulses. To find an approximation for re-sampling operation is the main study in this paper.

1. INTRODUCTION

It has been proposed that to increase the SNR without complicating the sensor circuit design, the even-odd sensor layout will be considered. However, the image processing systems available today does not take into count the strategy of the even-odd sampling layout.

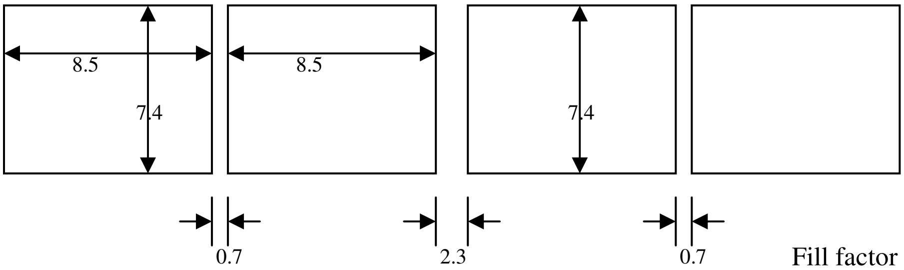
The rectangular shapes of pixel active area are studied in this analysis.[1] As Figure 1 shows, the design explores two

---

[1] May not the same as the layout of FS-5

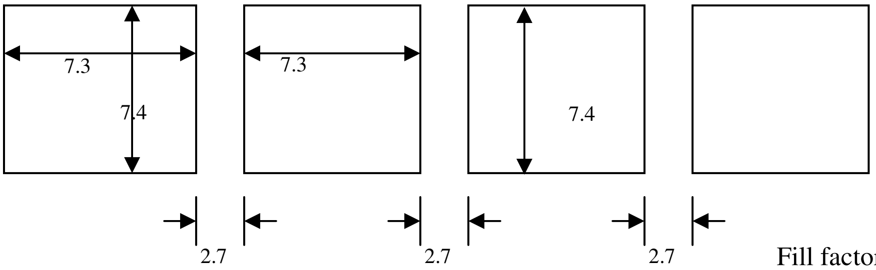
possible options: mirror-type and non-mirror-type pixel layouts. The mirror-type layout is designed for higher FF (61%), but it lefts non-equal spacing issue between the even and odd pixels. The non-mirror–type one is designed for the concept of equal spacing sampling interval between the even and odd pixels and results in lower FF(54%) .

Mirror–type pixel layout for PAN (pixel size is 10 um x 10 um)



Fill factor : 61%

Non-mirror–type pixel layout for PAN (pixel size is 10 um x 10 um)



Fill factor: 54%

Figure 1

To figure out the even-odd effects, Whittaker-Shannon interpolation formula was applied to construct signals for even-odd series pixels from equal-spaced signals [2]. The simulated signals are then used to evaluate the possible deformation arose from conventional image re-sampling algorithms. To further explore the impacts of even-odd effects on the static MTF, an ideal PSF signal is introduced and sampled with even-odd series. The static MTF is about 6×10^{-3} difference affected in high frequency range [2].

There are two possible approaches for ground image processing. A trade-off study is necessary. First of those is treating the even-odd sampling layout as the equal-spacing sampling scheme, and analyze the impacts of possible error introduced when applying the traditional re-sampling kernel. Secondly, deriving the perfect reconstruction algorithm and see if the computer resource required is manageable.

The article focus on second approach, and is organized as follows: We begin with a review of bandlimited interpolation theroy , leading up to Kohlenberg's resampling kernel for Reconstructing X(t) from non-uniform sampled data. Some test cases based on the simplified re-sampling kernel was demonstrated. Finally get conclusion.

2. THEORY OF BANDLIMITED INTERPOLATION

According to the sampling theory, given a band-limited continuous signal X(t), it is possible to reconstruct the signal X(t) from the discrete sampled $X(t_i)$, where t_i are equal spaced sampled ([1]). To reconstruct X(t) at any t, the sinc function is introduced according to sampling theory. For t_i extending to non-equal spaced sampled cased were presented in the last study [3] also.

2.1 Ideal Case (Equal Spacing, Uniform-Rate Sampling):

We review briefly the "analog interpretation" of sampling rate conversion [4] on which the present method is based. Suppose we have samples x(nTs) of a continuous absolutely integrable signal x(t), where t is time in seconds (real), n ranges over the integers, and Ts is the sampling period. We assume x(t) is bandlimited to ±W, where W =1/(2Ts) is the sampling rate. If X(ω) denotes the Fourier transform of x(t), i.e., X(ω) = $\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, then we assume X(ω) = 0 for | ω | ≥ 2 π W.

Consequently, Shannon's sampling theorem gives us that x(t) can be uniquely reconstructed from the samples x(nTs) via $\hat{x}(t) \equiv \sum_{n=-\infty}^{\infty} x(nT_s)h_s(t-nT_s)$ …………..(2.1a), where

$$h_s(t) \equiv \sin c(2Wt) \equiv \frac{\sin(2\pi Wt)}{2\pi Wt} \quad \text{……………………(2.1b)}$$

To resample x(t) at a new sampling rate 2W′s = 1/T′s, we need only evaluate Eq. (2.1a) at integer multiples of T′s. The sinc function can be seen as a hyperbolically weighted sine function with its zero at the origin canceled out. The name sinc function derives from its classical name as the sine cardinal (or cardinal sine) function.

2.2 Key Words Non-Ideal Case (Non-Equal Spacing):

Periodic sampling, introduced by Kohlenberg [3], is well established which considers
sampling sets as unions of cosets of one subgroup. The function is determined by its values at a set of points of density 2W, but the points consist of two similar groups with spacing 1/W, shifted with respect to each other.
Kohlenberg's paper derive the pth order sampling function, and prove that in the case of first order, sampling in equal spacing, the re-sampling kernel is sinc function, in the case of second order, can be applied to the unions of two subgroup, exactly applicable to the case of even and odd group.
Kohlenberg's formula:

$$f(t) = \sum_n [f(n/W)s(t-n/W) + f(n/W+k)s(n/W+k-t) \text{, ----- (Eq.2.2a)}$$

with $s(t) = \dfrac{\cos(2\pi(W_0+W)t-(r+1)\pi Wk) - \cos(2\pi(rW-W_0)t-(r+1)\pi Wk)}{2\pi Wt\sin((r+1)\pi Wk)}$

$$+\frac{\cos(2\pi(rW-W_0)t-r\pi Wk) - \cos(2\pi W_0 t - r\pi Wk)}{2\pi Wt\sin(r\pi Wk)}, \quad \text{----------------- (Eq.2.2b)}$$

the function is determined by its values at a set of points of density *2W,* but the points consist of two similar groups with spacing *1/W,* shifted with respect to each other.

3. RESAMPLING KERNAL & SPATIAL FINITE APPROXIMATIONS

For that the reconstruction process reconstructs X(t) at any t by convoluting the sinc function & the discrete sampled $X(t_i)$, and both of them are infinite-supported in spatial domain, spatial finite approximations were introduced for image reconstruction, of them, the nearest neighbor, bi-linear, and 2-D cubic convolution approaches are commonly available in current image processing systems. ([6])

To achieve the best reconstruction results with limited calculation steps, 2-D cubic convolution approach is adopted in most systems. Though the cubic-spline approach performs marginally better then cubic convolution ([5]), its calculation relies on all the data available. To find X(t) at any t, the calculation use information from all $X(t_i)$. As a result, current image processing systems do not adopt the approach. Mainly, the computer resources required for this approach are beyond the manageable means.

Cubic convolution is the 3-order polynomial simplification of sinc function, it is applicable with the assumption of input signal is equal spacing; Our goal is to find a suitable convolution kernel applicable for non-equal spacing case, KOHLENBERG 's 2^{nd} order reconstruction is quite complex (Eq. 2.2b), and requires summation of infinite series. For our application, we can rewrite Eq. 2.2b as Eq. 3.1 by setting $W_0 = 0$, and r = 0:

$$s(t) = \frac{\cos(2\pi Wt - \pi Wk) - \cos(\pi Wk)}{2\pi Wt \sin(\pi Wk)} \quad \text{.... (Eq. 3.1)}$$

Before deriving a spatial-limited convolution kernel, we study some test cases by using Eq. 3.1 and set band width W = 360 deg/ 2*2m, phase shift k = 0.5-0.04. The following figure describes the meaning of k and W.

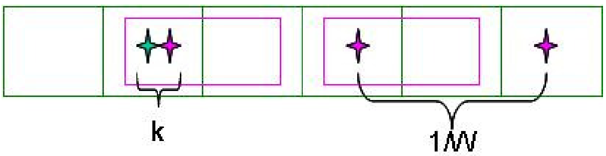


Figure 2. k and W in (Eq. 3.1)

4. TEST CASES

Use Simulated Ideal PSF Signal

To examine the resample kernel (Eq. 3.1) for non-uniform sampling group, we take a simulated ideal PSF (Point Spread Function) as a test object. The experience comprises the following steps:

Step 1: Use truncated Gaussian function to generate PSF signal, with array size: 60 x 1. Figure 3.2a shows the simulated signal f(t).

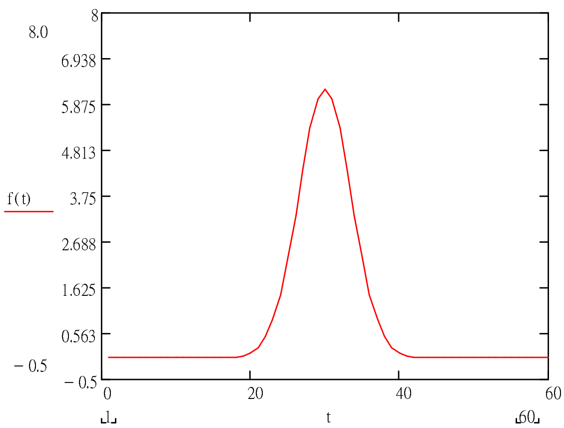


Figure 3.2a

Step 2: Reconstruct the discrete PSF by using the following re-sampling kernel simplified from KOHLENBERG 's formula to validate the reconstruction process:

$$s(t) := \frac{\cos(2\cdot\pi\cdot W\cdot t - \pi\cdot W\cdot k) - \cos(\pi\cdot W\cdot k)}{2\cdot\pi\cdot W\cdot t\cdot\sin(\pi\cdot W\cdot k)} \quad \text{....... (Eq. 4.1a)}$$

$$f(t) := \sum_n \left(f1\left(\frac{n}{W}\right)\cdot s\left(t - \frac{n}{W}\right) + f1\left(\frac{n}{W} + k\right)\cdot s\left(\frac{n}{W} + k - t\right)\right) \quad \text{....... (Eq. 4.1b)}$$

With W = 360 deg/ 2*2m, and k = 0.5-0.04

By examining the difference between each reconstructed elements and Figure 3.2b shows the difference between them, the process is validated in the first stage.

Step 3: Construct non-equal spacing sampled PSF by using the following equations, a non-equal sampling sequence result is obtained as shown in Figure 3.2c.

$$f2(n) := \left[\begin{array}{ll} \sum_k [f1(k)\cdot \text{sinc}[n - (k - 0.04)]] & \text{if} \quad \text{mod}(\text{trunc}(n), 2) = 0 \\ \sum_k [f1(k)\cdot \text{sinc}[n - (k + 0.04)]] & \text{if} \quad \text{mod}(\text{trunc}(n), 2) = 1 \end{array} \right] \quad \text{....... (Eq. 4.1c)}$$

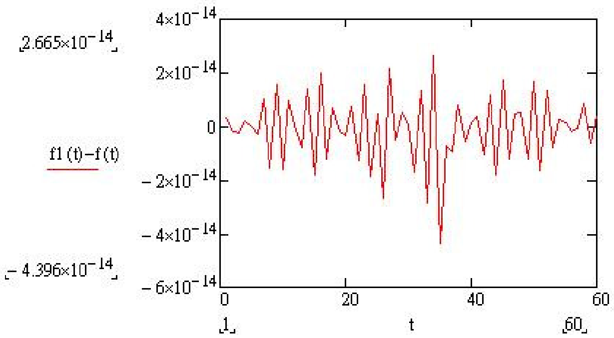


Figure 3.2b: difference of PSF signal and re-constructed PSF signal

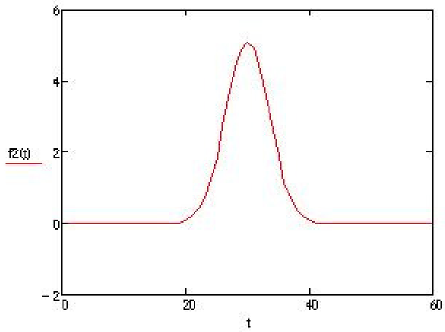


Figure 3.2c: constructed non-equal spacing PSF signal

Step 4: Repeat the procedure in step 2, applying the (Eq. 4.1d) for the non-equal spacing sampled PSF signal from step 3, the difference between two signal is shown in figure 3.2d.

$$f3(t) := \sum_n \left(f2\left(\frac{n}{W}\right)\cdot s\left(t - \frac{n}{W}\right) + f2\left(\frac{n}{W} + k1\right)\cdot s\left(\frac{n}{W} + k1 - t\right)\right) \quad \text{....... Eq. (4.1d)}$$

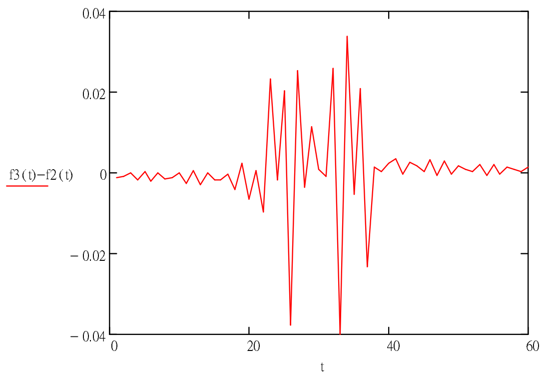


Figure 3.2d: difference of non-equal spacing PSF signal and re-constructed non-equal spacing PSF signal

5. CONCLUSIONS & FUTURE WORKS

This work demonstrates the applicability of re-sampling kernel for the non-equal spacing between even and odd sensor pixel arrays. With the test results shown in section 4.1, the error introduced by re-sampling kernel is quite small (0.04 DN). Our work proved this equation could be used for image resampling operation.

In the near future, we are going to derive the simplified kernel with finite summation operations, more simulated non-equal spacing images will be generated for verification purpose.

References

[1] C. E. Shannon, "Communication in the presence of noise", Proc. Institute of Radio Engineers, vol. 37, no. 1, pp. 10–21, Jan. 1949. Reprint as classic paper in: *Proc. IEEE*, vol. 86, no. 2, (Feb. 1998)

[2] Cynthia Liu, Nai-Yu Chen, Non-equal spacing CMOS sensor impact on response between even and odd pixels, Oct. 2010, *SPIE Asia Conference*

[3] Arthur Kohlenberg, "Exact Interpolation of Band-Limited Functions", Journal Of Applied Physics Volume 24. Number 12, December. 1953

[4] R. Crochiere and L. R. Rabiner, *Multirate Digital Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall, 1983.

[5]. Eric H. W. Meijering, Wiro J. Niessen, Max A. Viergever, Piecewise Polynomial Kernels For Image Interpolation: A Generation Of Cubic Convolution.

[6]. Robert G. Keys, Cubic Convolution Interpolation for Digital Image Processing, IEEE Transactions On Acoustics, Speech, And Signal Processing, Vol. ASSP-29, No. *6,* December 1981