MULTI-BAND IMAGE CLASSIFICATION USING MEMBERSHIP FUNCTIONS

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ABSTRACT: We propose a method for remotely sensed multi-band image classification using membership function. Our aim is to classify the image as classes with use-defined number from a prior knowledge of remote sensing, and every point is finally labeled as the class with the highest value of membership function. This classification is reduced to a minimization problem of a functional whose arguments are membership functions. The minimization problem is solved via iteration with the initial value from the classification result of fuzzy C-means. Our method refines the result of fuzzy C-means and produces a smoother and less cluttered classification. Two novelties compared with traditional membership methods are in this paper. First, unconstrained functional is used. Constraints are added in the literature since membership functions need to be positive and with sum equal to one, which is avoided here by variable substitution. Second, intermediate variables are introduced so that a big complicated functional is separated as three relatively easy functionals that can be solved with fast speed. The experimental results from Google Map and Quickbird images show the validity of this approach.

1. INTRODUCTION

Image classification is an important topic in remote sensing applications, and a variety of classification methods for remotely sensed images have been studied (Lu and Weng, 2007). Roughly speaking, classification is to label every pixel into a certain class according to some criteria. Although various classification methods have already been proposed, it is still hard to find an algorithm suitable for all purposes. Thus developing a fast, robust and accurate classification technique remains a challenging task.

In this paper we develop a classification algorithm in which human interference is as little as possible. We only need to specify the number of classes to be classified, which is determined by the purpose of classification and a prior knowledge of the region in the satellite image. Our method uses membership function: for every point, its membership function, which is the probability that it belongs to each class, is calculated. The point is finally labeled as the class with the highest value of membership function. That is to say, our method is an unsupervised classification method based on membership function.

Traditional unsupervised classification methods, such as K-Means and IsoData, only consider the digital number (DN) value of a pixel, and neglect its geometric property (e.g. neighboring pixels). Such methods tend to produce a cluttered classification result. In real applications, we often place more importance on the macro patterns in the classification result, and not on isolated or scattered points. For example, we may wish to determine that a particular region lies in a body of water, without distracting our attention to the ships in the region. In this case, we may view the pixels representing the ships as “outliers” that are not of essential importance. The purpose of this paper is to propose a method that refines traditional methods to produce a smoother and less cluttered classification.

In our algorithm, geometric information is taken into account. We want the result to have some kind of smoothness; for instance isolated point(s) should be wiped out as much as possible in the final result. We will conduct the classification by proposing an energy functional and solving its minimizer. The minimization process is conducted via iteration: the result of traditional unsupervised classification is used as our initial value, membership functions is then calculated by iteration, and final membership functions represent the classification result.

The rest of the paper is organized as follows. Section 2 describes some background knowledge. Section 3 states the methodology. Section 4 is the result and discussion part. Section 5 concludes the paper.

2. BACKGROUND

2.1 Membership functions

A membership function (Zadeh, 1965) $M_i(x)$ represents the possibility that the point x belongs to the Class i, $i = 1, \ldots, K$. Thus, membership functions should follow the constraints below:

$$0 \leq M_i \leq 1 \qquad \text{and} \qquad \sum_{i=1}^{K} M_i = 1. \tag{1}$$

Apparently, the degree of freedom of M_i is $K-1$, which means M_i can be represented by $K-1$ free variables u_i, $i = 1, \cdots, K-1$. We now describe a way connecting M_i and u_i.

For instance, suppose $K = 8$. Then 8 membership functions can be expressed as

$$\begin{aligned}
M_1 &= \cos^2(u_1)\cos^2(u_2)\cos^2(u_4),\\
M_2 &= \cos^2(u_1)\cos^2(u_2)\sin^2(u_4),\\
M_3 &= \cos^2(u_1)\sin^2(u_2)\cos^2(u_5),\\
M_4 &= \cos^2(u_1)\sin^2(u_2)\sin^2(u_5),\\
M_5 &= \sin^2(u_1)\cos^2(u_3)\cos^2(u_6),\\
M_6 &= \sin^2(u_1)\cos^2(u_3)\sin^2(u_6),\\
M_7 &= \sin^2(u_1)\sin^2(u_3)\cos^2(u_7),\\
M_8 &= \sin^2(u_1)\sin^2(u_3)\sin^2(u_7),
\end{aligned} \tag{2}$$

where u_i is free variable. Via this method, $\{M_i\}$ satisfy the membership constraints (1) automatically. In this way, to find the constrained solution of M_i is reduced to find the solution of free variable u_i.

2.2 Fuzzy C-Means

We now introduce Fuzzy C-Means (FCM), as this method will be used to obtain the initial value of our classification.

FCM (Bezdek, 1981) is an unsupervised classification method. As an improvement of K-Means, FCM uses an idea of "soft" classification. Given K classes to be classified, for every pixel, FCM computes its membership functions; then the pixel is labeled as the class with highest membership function. To be more specific, FCM minimizes the following C-Means functional:

$$J = \sum_{i=1}^{N}\sum_{j=1}^{K} M_{ij}^m \|x_i - c_j\|^2,$$

where x_i is the i-th data point in the d-dim space, c_j is cluster center of Class j, M_{ij} is the membership function of x_i in Class j, m is any real number greater than 1, K is the number of classes to be classified, and N is total number of data. Here $\| \cdot \|$ can be any form of norm.

This minimization problem can be solved by iteration. In k-th iteration, the membership M_{ij} and class center c_j can be expressed by

$$M_{ij}^{(k)} = \frac{1}{\sum\limits_{n=1}^{K} \left(\frac{\|x_i - c_j\|}{\|x_i - c_n\|} \right)^{2/(m-1)}} \qquad \text{and} \qquad c_j^{(k)} = \frac{\sum\limits_{i=1}^{N} M_{ij}^m \cdot x_i}{\sum\limits_{i=1}^{N} M_{ij}^m}.$$

This iteration will terminate when the stopping criteria are satisfied.

In this paper we use the code downloaded from Fuzzy Clustering and Data Analysis Toolbox (available at http://www.fmt.vein.hu/softcomp/fclusttoolbox) to implement FCM.

Figure 1a and 1b shows an example of FCM classification. Figure 1a is an input image downloaded from Google Map, and Figure 1b is its FCM result. Here the image is classified as 8 classes, where 8 is specified according to user's requirement. We can see that the result is cluttered in some places, especially in the plantation area. This is caused by the fact that FCM is a point-wise approach, which only considers the pixel values and neglects geometric information. In our algorithm, the FCM result is used as an initial value for iteration, and we will refine it to produce a smoother and less cluttered classification.

3. METHODOLOGY

In this section our aim is to classify a d-band image $I(x)$ in domain Ω as K classes. Without loss of generality, we suppose $K = 8$.

3.1 Proposed functional

At beginning, we first propose the following functional:

$$E(\{M_i\}, \{c_i\}) = \sum_{i=1}^{K} \int_\Omega |\nabla M_i| + \lambda \sum_{i=1}^{K} \int_\Omega \|I - c_i\|^2 \cdot M_i, \tag{3}$$

where $\{M_i\}$ are the membership function; c_i is a d-dim vector representing mean value in Class i, and the component of c_i is the mean value of Class i in each band; and λ is a balancing parameter.

The functional (3) could be explained as follows. The first term is a regularization term, guaranteeing the smoothness of the membership function. The second term is a fidelity term, guaranteeing that the difference of pixel values is controlled in each class.

Please note that functional (3) is a constrained functional as M_i should satisfy membership constraints (1). Therefore we introduce free variable u_i in (2) to substitute M_i, and change (3) to be an unconstrained functional:

$$E(\{u_i\}, \{c_i\}) = \sum_{j=1}^{K-1} \int_\Omega |\nabla u_j| + \lambda \sum_{i=1}^{K} \int_\Omega d_i \cdot M_i(u_1, \cdots, u_{K-1}) \tag{4}$$

where $d_i = \|I - c_i\|^2$. Note that the first term on the right hand side is a regularization term for free variables.

For the sake of easy computation, the functional (4) is further turned into:

$$E(\{u_i\}, \{v_i\}, \{c_i\}) = \int_\Omega \sum_{j=1}^{K-1} |\nabla v_j| + \frac{1}{2\theta} \int_\Omega \sum_{j=1}^{K-1} |v_j - u_j|^2 + \lambda \sum_{i=1}^{K} \int_\Omega d_i \cdot M_i, \tag{5}$$

where v_i is an intermediate variable; θ is another balancing parameter.

The purpose of introducing v_i is that we can apply operator/variable splitting method to (5). The term $\int_\Omega \sum_{j=1}^{K-1} |v_j - u_j|^2$ makes sure the closeness between u_i and v_j.

As functional (5) is non-convex, we can only achieve a local minimum. Thus, we use the result of FCM as the initial value for iteration to guarantee that our result is near to a reasonable solution.

3.2 Detailed steps

To minimize (5), we separate the problem into 3 sub-problems.

When updating one parameter, all other parameters are considered constants during the iteration.

In each iteration,

1. Update v_i:

This is a standard ROF model (Rudin et al., 1992). This model can be solved using various methods.

2. Update c_i:

$$c_i = \frac{\int_\Omega I \cdot M_i}{\int_\Omega M_i};$$

3. Update u_i:

u_i can be obtained by the Euler-Lagrange equation with respect to u, which can be reduced to a series of triangular equations:

$$\frac{1}{\theta}(u_j - v_j) - \lambda \sin(2u_j) r_j = 0, \qquad (j = 1, \cdots, K-1),$$

where

$$\begin{aligned}
r_1 &= d_1 \cos^2(u_2)\cos^2(u_4) + d_2 \cos^2(u_2)\sin^2(u_4) \\
&\quad + d_3 \sin^2(u_2)\cos^2(u_5) + d_4 \sin^2(u_2)\sin^2(u_5) \\
&\quad - d_5 \cos^2(u_3)\cos^2(u_6) - d_6 \cos^2(u_3)\sin^2(u_6) \\
&\quad - d_7 \sin^2(u_3)\cos^2(u_7) - d_8 \sin^2(u_3)\sin^2(u_7) \\
r_2 &= d_1 \cos^2(u_1)\cos^2(u_4) + d_2 \cos^2(u_1)\sin^2(u_4) \\
&\quad - d_3 \cos^2(u_1)\cos^2(u_5) - d_4 \cos^2(u_1)\sin^2(u_5) \\
r_3 &= d_5 \sin^2(u_1)\cos^2(u_6) + d_6 \sin^2(u_1)\sin^2(u_6) \\
&\quad - d_7 \sin^2(u_1)\cos^2(u_7) - d_8 \sin^2(u_1)\sin^2(u_7) \\
r_4 &= (d_1 - d_2)\cos^2(u_1)\cos^2(u_2), \\
r_5 &= (d_3 - d_4)\cos^2(u_1)\sin^2(u_2), \\
r_6 &= (d_5 - d_6)\sin^2(u_1)\cos^2(u_3), \\
r_7 &= (d_7 - d_8)\sin^2(u_1)\sin^2(u_3).
\end{aligned}$$

This kind of triangular equation can be easily solved by Newton's method.

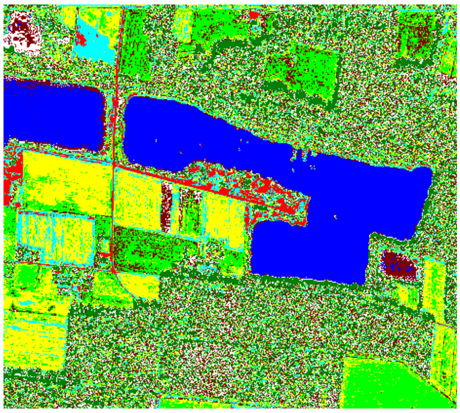
The iteration terminates when stopping condition is satisfied. From experience, iteration number 30 is sufficiently good for most cases.

4. RESULT AND DISCUSSION

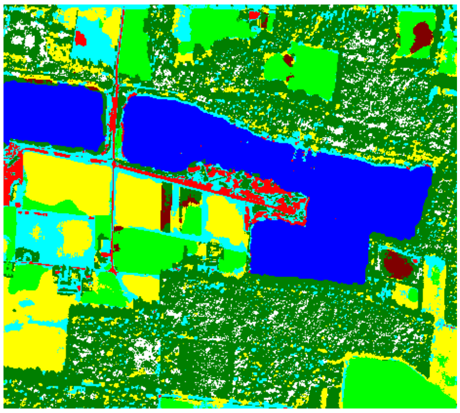
We use two examples to show the results. Example 1 is a RGB image downloaded from Google Map. Example 2 is a quickbird 4-band image downloaded from http://glcf.umiacs.umd.edu/data/quickbird/.

Figure 1 shows the classification result of Example 1. Figure 1a is the input image; Figure 1b is the classification result of 8 classed using FCM, which is used as the initial value for our algorithm. Figure 1c is our result. Parameter used is the algorithm is: $\lambda = 0.0005$; $\theta = 0.1$; number of iteration = 30; processing time is 37 seconds for a CPU of 1.83 GHz with 3G RAM.

(a) Input image



(b) FCM result (8 classes)



(c) Result of our algorithm

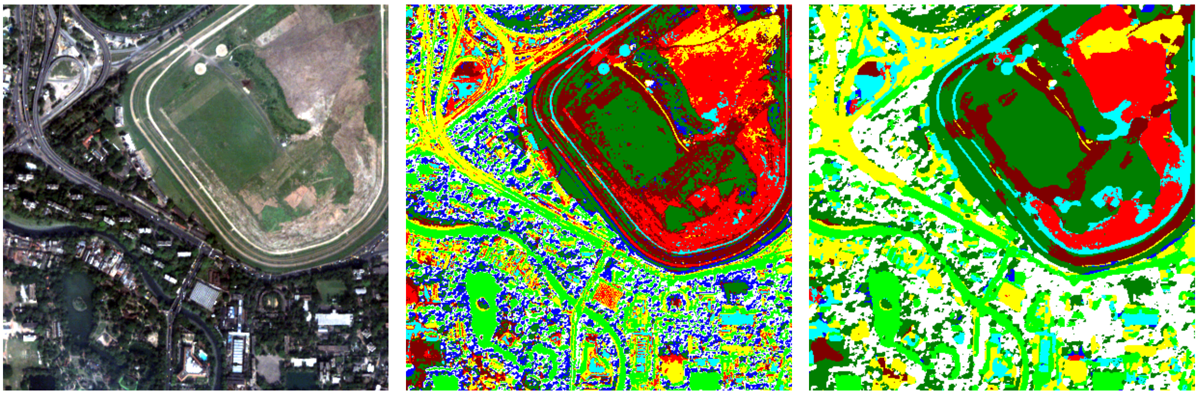
Figure 1: Classification result of a Google Map image

Figure 2 shows the classification result of Example 2. Figure 2a is the input image (color display: red: band 3; green: band 2 and blue: band 1); Figure 2b is the classification result of 8 classed using FCM, which is used as the initial value for our algorithm. Figure 2c is our result. Parameter used is the algorithm is: $\lambda = 0.0005$; $\theta = 0.1$; number of iteration = 30; processing time is 33 seconds for a CPU of 1.83 GHz with 3G RAM.

In general the results are satisfactory. Our method refines the result of fuzzy C-means and produces a smoother and less cluttered classification. Several small areas are combined, with inside isolated points moved.

We should point out two things worth mentioning in our algorithm. First, the solution of our algorithm is a local minimum, not a global one; which means that our result is only a refinement of the FCM result (i.e., the two results are close). For instance, if FCM provides totally wrong classification, it is hard to correct via our algorithm. This can be observed from Figure 2. There is a main road across the image from upper left to the center. Unfortunately the road is misclassified as two classes (yellow and green) in the FCM, and it still remains two classes in the final result.

Second, in our examples, the number of classes is set as $K = 8$, which is in the category of 2^n. This is a relatively easy case to covert K membership functions to $K - 1$ free variables. If K does not belong to



(a) Input image (b) FCM result (8 classes) (c) Result of our algorithm

Figure 2: Classification result of a Quickbird image

that category, for example, $K = 5$, we can use the same idea to express the 5 membership functions by

$$\begin{aligned} M_1 &= \cos^2(u_1)\cos^2(u_2)\cos^2(u_4), \\ M_2 &= \cos^2(u_1)\cos^2(u_2)\sin^2(u_4), \\ M_3 &= \cos^2(u_1)\sin^2(u_2), \\ M_4 &= \sin^2(u_1)\cos^2(u_3), \\ M_5 &= \sin^2(u_1)\sin^2(u_3) \end{aligned}$$

with 4 free variables. Here we use the idea of dichotomy. That means that if we want to separate one class into two classes, we only need to add one free variable using the fact that the sum of cosine squared and sine squared equals one.

5. CONCLUSION

We have proposed a method for remotely sensed multi-band image classification using membership function. This is done via minimizing an energy functional. The minimization problem is solved via iteration with the initial value from the classification result of fuzzy C-means. Our method refines the result of fuzzy C-means and produces a smoother and less cluttered classification. The experimental results from Google Map and Quickbird images show the validity of this approach.

Several improvements can be done in the future. For instance, to make the classification more accurate, we can convert this algorithm to a supervised or guided classification, which will use training samples instead of FCM result as initial values.

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