# GEOMETRY ANALYSIS BASED GENETIC ALGORITHM FOR SATELLITE IMAGING SCHEDULING

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KEY WORDS: Satellite Imaging Scheduling, Earth-satellite geometry, Genetic algorithm.

**ABSTRACT:** In this paper, a novel method is proposed to facilitate the genetic algorithm for the problem of satellite imaging scheduling. First, geometry analysis is applied to target assignment and coordination for each satellite track. Second, a general formula is derived to estimate the degree of availability for each scenario specified by the new iterated solution. Multi-satellite scheduling scenario is performed to verify the proposed algorithm. Simulation results show that the proposed algorithm avoids large amount of calculation and provides high quality solution to help generating better solutions within a short period of time. Moreover, the efficiency compared with traditional genetic algorithm is also discussed.

# 1. INTRODUCTION

Satellite imagery has become an increasingly important in disaster relief and land monitoring. The purpose of satellite imaging scheduling is to make the most of limited satellite resources to satisfy the requirement of resolution and delivery term for each ground target image. The difficulty lies in the fact that the coordination between satellites and observation requests is a very complicated problem, which made it impractical to develop an algorithm for the optimal scheduling purpose. A considerable amount of theoretical work has been carried out on imaging scheduling for a single or multi-satellite scheme [1][2]. It has been found that the genetic algorithm outperforms this well known NP-hard problem [3][4]. However, the generated new solution still needs to be verified under various constraints to make sure its availability for each iteration step. As a result, fast task assignment and availability decision are the key factors for a genetic algorithm to quickly find the high quality solution.

Based on spherical geometry, a novel method is proposed to facilitate the genetic algorithm for the problem of satellite imaging scheduling. Instead of considering satellite as a single machine in most of the literatures, we consider a mission track as a single machine. Thus a satellite consists of many mission tracks in the scheduling time horizon, and mission track coordination is necessary for both single satellite and multiple satellites scheduling problems. Given the knowledge of selected satellites and a set of observation targets, the scheduling planning is considered as uncorrelated parallel machine scheduling problem. An Earth-satellite geometry analysis is performed to develop an algorithm to determine virtual ascending/descending nodes for a specific ground target, which facilitates target assignment and mission track coordination without blind search. After target assignment in a specific mission track, truncated Cauchy distribution based genetic algorithm is proposed to solve the scheduling problem on this fixed scenario.

# 2. EARTH-SAELLITE GOMETRY

In order to fast assignment of ground target candidates for each satellite track, a spherical geometry analysis is performed as follows. Figure 1 shows earth-satellite geometry in Earth-central Fixed (ECF) coordinate system. By Napier's rule for right spherical triangles [5], the track of subsatellite points (SSP) is a great circle arc and can be expressed as

$$\tan\phi = \tan i \cdot \sin\left(\lambda - \lambda_A\right) \tag{1}$$

$$\sin\phi = \sin i \cdot \sin u \tag{2}$$

where  $\phi$  and  $\lambda$  are the latitude and longitude of SSP,  $\lambda_A$  is the ascending point longitude of the satellite groundtrack, *i* is the satellite orbit inclination, and *u* is the earth-central angle relative to the arc-length distance between the ascending point *N* and the subsatellite point *S*. Let  $\omega_S$  be the angular velocity of satellite rotating around the Earth, by rewriting (2) as a function of time *t*, we have

$$\phi(t) = \sin^{-1}\left(\sin i \cdot \sin \omega_s \left(t - t_0\right)\right) = \sin^{-1}\left(\sin i \cdot \sin\left(\frac{2\pi(t - t_0)}{T_s}\right)\right)$$
(3)

where  $t_0$  is the epoch time the satellite fly over the ascending point *N*. In the following analysis, we consider *N* is the reference point and assume  $t_0 = 0$  to simplify our analysis.



Figure 1. Earth satellite Geometry

From (3) we see that, given a SSP latitude  $\phi$  in ascending or descending period, the time spend of satellite passing through the ascending node *N* can be estimated by

$$t = \begin{cases} \frac{T_s}{2\pi} \sin^{-1} \left( \frac{\sin \phi}{\sin i} \right) & \text{ascending period} \\ \frac{T_s}{2} - \frac{T_s}{2\pi} \sin^{-1} \left( \frac{\sin \phi}{\sin i} \right) & \text{descending period} \end{cases}$$
(4)

Now, Let us consider the effect of Earth rotation on SSP track. Due to Earth rotation, the SSP track cannot be considered as a great circle arc. Actually, according to geometry analysis, the angular velocity of the satellite relative to the Earth in ECF coordinate system,  $\omega_F$ , can be expressed as a function of latitude  $\phi$ , given by [6]

$$\omega_F^2 = \omega_S^2 + \omega_E^2 \cos^2 \phi - 2\omega_S \omega_E \cos i$$
  
=  $(\omega_S - \omega_E \cos \phi)^2 + 2\omega_S \omega_E (\cos \phi - \cos i)$  (5)

And the procession of SSP longitude at time t due to Earth rotation can be obtained by

$$\Delta\lambda_E(t) = -\frac{2\pi t}{T_E} \tag{6}$$

where  $T_E$  is the rotation period of the Earth. As a result, by (1) and (6), we have

$$\lambda(t) = \lambda_A + \sin^{-1} \left( \frac{\tan \phi(t)}{\tan i} \right) + \Delta \lambda_E(t)$$

$$= \lambda_A + \sin^{-1} \left( \frac{\tan \phi(t)}{\tan i} \right) - \frac{2\pi t}{T_E}$$
(7)

In order to estimate the position of SSP at any time by a given SSP, assume that  $Q(\phi_q, \lambda_q)$  is a known SSP position at time *t* in ascending period. By (7), the ascending node latitude  $\lambda_A$  can be derived by

$$\begin{split} \lambda_A &= \lambda_q - \sin^{-1} \left( \frac{\tan \phi_q}{\tan i} \right) + \frac{2\pi t}{T_E} \\ &= \lambda_q - \sin^{-1} \left( \frac{\tan \phi_q}{\tan i} \right) + \frac{2\pi}{T_E} \cdot \frac{T_S}{2\pi} \sin^{-1} \left( \frac{\sin \phi_q}{\sin i} \right) \\ &= \lambda_q - \sin^{-1} \left( \frac{\tan \phi_q}{\tan i} \right) + \frac{1}{M_0} \sin^{-1} \left( \frac{\sin \phi_q}{\sin i} \right) \end{split}$$
(8)

where  $M_0 = T_s / T_E$  is the mean motion of the satellite.

However, in the case of Q in the period of descending,  $\lambda_A$  should be obtained by deriving descending node longitude. Let  $t_0 = 0$  is the time epoch of satellite at descending point, by geometry analysis the time epoch of satellite at Q is given by

$$t = \frac{T_s}{2\pi} \sin^{-1} \left( \frac{\sin \phi}{\sin i} \right) \tag{9}$$

Substitute (9) into (7), we have

$$\lambda(t) = \lambda_D - \sin^{-1}\left(\frac{\tan\phi(t)}{\tan i}\right) - \Delta\lambda_E(t) = \lambda_D - \sin^{-1}\left(\frac{\tan\phi(t)}{\tan i}\right) + \frac{2\pi t}{T_E}$$
(10)

Since  $\lambda(t) = \lambda_{q}$ , we have

$$\begin{split} \lambda_{D} &= \lambda_{q} + \sin^{-1} \left( \frac{\tan \phi_{q}}{\tan i} \right) - \frac{2\pi t}{T_{E}} \\ &= \lambda_{q} + \sin^{-1} \left( \frac{\tan \phi_{q}}{\tan i} \right) - \frac{2\pi}{T_{E}} \cdot \frac{T_{S}}{2\pi} \sin^{-1} \left( \frac{\sin \phi_{q}}{\sin i} \right) \\ &= \lambda_{q} + \sin^{-1} \left( \frac{\tan \phi_{q}}{\tan i} \right) - \frac{1}{M_{0}} \sin^{-1} \left( \frac{\sin \phi_{q}}{\sin i} \right) \end{split}$$
(11)

As a consequent, the ascending node longitude can be obtained by

$$\lambda_{A} = \lambda_{D} - \pi - \Delta \lambda = \lambda_{D} - \pi + \frac{2\pi t}{T_{E}}$$

$$= \lambda_{D} - \pi + \frac{2\pi}{T_{E}} \frac{T_{s}}{2} = \lambda_{D} - \pi + \frac{\pi}{M_{0}} = \lambda_{D} - \pi \left(1 - \frac{1}{M_{0}}\right)$$
(12)

#### 3. TARGET ASSIGNMENT

Most remote sensing satellites use sun-synchronous orbit designing to allow revisiting the same place periodically. Let  $\lambda_{A_i}$  and  $\lambda_{D_i}$  be the ascending node and descending longitude of *i*-th satellite track, respectively.

Given knowledge of different tasking requirements for hundreds of ground targets around the world, the target assignment and coordination is one of the most important steps for satellite imaging scheduling, which determines how many mission tracks can take photo of a given target and the preferred order of these available mission tracks. An optimal scheduling result may have finally achieved after lots of tries. In order to facilitate the efficiency of target assignment and coordination, the derived ascending/descending estimation is employed to obtain the virtual ascending/descending node and then the neighboring mission track ascending/descending nodes are considered to be the available tracks. To be specific, consider a remote sensing satellite orbiting around the Earth at an inclination of 99.1° and an altitude of 891 km. Figure 2 shows the ascending/descending nodes matches the real satellite tracks. While  $Q_2(22.75^\circ N, 125.2^\circ E)$  is not a SSP, which is considered as a ground targets, the obtained virtual ascending/descending nodes determine the nearby ascending/descending satellite tracks are the candidates for imaging  $Q_2$ .



Figure 2. Ascending/Descending node prediction results (a) matches real satellite track if Q1 is a SSP (b) find virtual nodes if Q2 is not a SSP.

#### 4. MISSION TRACK IMAGING SCHEDULING

In this paper, a truncated Cauchy distributed genetic algorithm (TCDGA) is developed to solve the problem of mission track imaging scheduling. Figure 3 shows the flowchart of this proposed TCDGA algorithm. Given knowledge of satellite mission track SSPs and elevation mask of each assigned target, the time duration that a ground target can be imaged by a satellite, defined as time of opportunity (TOW), and the trimmed version of TOW, named TTOW, can also be obtained according to the constraints, such as satellite view angle and thermal power. Truncated Cauchy distribution is employed to determine the best imaging time epoch quickly according to the scenario. Finally, genetic algorithm is employed to find the near-optimal scheduling according to the specified objective function.



Figure 3. Flowchart of TCDGA Algorithm

# 4.1 TTOW and TCPDF

The time duration that a ground target  $G_n$  can be imaged by a satellite is regarded as time of opportunity (TOW)[7], defined as

$$W_n = \left\{ q_n \in N \middle| t_{S_n} \le q_n \le t_{E_n} \right\}$$
(13)

where  $q_n$  is the imaging time of  $G_n$ ;  $t_{S_n}$  and  $t_{E_n}$  are the start and end time of this TOW. When the previous imaging time for target  $G_{n-1}$ ,  $q_{n-1}$ , is determined, the earliest imaging time for the current target  $G_n$  may later then  $t_{S_n}$  due to maneuver constraint. As a result, we obtain a trimmed version of TOW (TTOW) for  $G_n$ , given by

$$TW_n = \left\{ q_n \in N \middle| t_{S_n} + \Delta_n \le q_n \le t_{E_n} \right\}, \ \Delta_n > 0$$
(14)

where

$$g_{n} + \Delta_n = q_{n-1} + P_{n-1} + \Delta t_{n-1,n}$$
(15)

 $P_{n-1}$  is the processing time for  $G_n$ ;  $\Delta t_{n-1,n}$  is the required maneuver time duration from  $G_{n-1}$  to  $G_n$ . Note that the use of TTOW greatly reduces search time for  $q_n$ , especially in the condition of target congestion, which leads to TOWs overlap each other seriously.

In order to further improve the quality of  $q_n$ , we use truncated Cauchy probability distribution function (TCPDF) to enhance the probability of favorite imaging time. Consider a TTOW with  $t_{\min} = t_{S_n} + \Delta n$ ,  $t_{\max} = t_{E_n}$ , and  $t_0 = (t_{S_n} + t_{E_n})/2$ , we have

$$f_{T_c}(t;t_0,\gamma,t_{\min},t_{\max}) = \frac{1}{\tan^{-1}\left(\frac{t_{\max}-t_0}{\gamma}\right) - \tan^{-1}\left(\frac{t_{\min}-t_0}{\gamma}\right)} \cdot \frac{\gamma}{\gamma^2 + (t-t_0)^2}$$

where  $\gamma$  is the parameter which specifies the overlap scale of adjacent TOW. Figure 4 shows two different TCPDF curves, by taking  $f_{T_c}(t;100,10,70,130)$  and  $f_{T_c}(t;100,10,120,180)$  as an example.



Figure 4. Two different TCPDF curves

#### 4.2 Genetic algorithm

Genetic algorithm is employed to find a near-optimal solution for specified scenarios. Consider a mission track attempts to imaging N targets with trimmed opportunity window  $TW_n$ , n = 1..N. First, a number of K individual, or chromosome, is generated as an initial population. Each individual contains the selected imaging time for each target, we have

$$S = \{s_i, i = 1, 2, \cdots, K\}.$$
(16)

$$Y_{i} = \{q_{i}^{(i)}, j = 1, 2, \cdots, N\}$$
(17)

Note that the imaging time is determined by the proposed time selection algorithm. Further, iterated crossover and mutation procedures are performed to reproduce new generations with better performance. Figure 5 depicts the detailed procedures for the employed genetic algorithm. Each step is described as follows:

- A) Generate *K* individuals as initial population.
- B) Rank the initial population according to the specified objective function and then choose the first K/2 individuals as parent population.
- C) In parent population, randomly select 2 individuals for crossover to reproduce 2 offsprings, until *N* offsprings reproduced. Here an iteration is completed.
- D) Mix parent and offspring population as a population. Rank this 2*N*-individual population according to the specified objective function and then choose the first N individuals as parent population.
- E) Repeat the procedure like C to obtain additional iteration.
- F) Repeat the procedure like D to obtain new parent population.
- G) Repeat E and F, until user-specified iterations are achieved. The first individual in the ranked final parent population is chose as the near-optimal solution.



Figure 5. Crossover mechanism

# 5. SIMULATION RESULTS

A computer simulation was performed to validate the performance of the proposed algorithm. Consider 20 random generated targets are planned to be imaged by 4 satellites. The mission tracks for SAT-1 and SAT-3 are

descending tracks due to the imaging tasks are planned during the satellite's motion from the North to the South, while SAT-2 and SAT-4 are ascending tracks. The SSPs of each track for each satellite are predicted by SGP4 orbital simulation model. The objective function is designed to be maximum image resolution, which means each target is desired to be imaged by satellite in a view angle as small as possible. By the proposed geometry analysis, Figure 6 shows the initial target assignment result, in which the preferred tracks are listed according to the order of distance from the target to the satellite track.



Figure 6. Initial target assignment result

From Figure 6 we see that the No.3 track of SAT-1 should take more photos than other tracks to satisfy the requirement of objection function. However, as G11 to G15 are crowded, SAT-1 could not complete the planned task due to maneuver constraint. As a result, targets should be reassigned until an available solution is obtained. Experiments show that the proposed algorithm for the mission track scheduling, which based upon truncated Cauchy distribution function and genetic algorithm, can accelerate the solution finding test in about seconds for each mission track by a Pentum-4 level computer.

# 6. CONCLUTIONS

We develop an algorithm for multi-satellite imaging scheduling. Based on geometry analysis, target assignment and mission track coordination is achieved in a short period of time, which facilitates genetic algorithm to find the near-optimal solution with fast time and good quality. The concept of trimmed time of opportunity window is used to reduce search time for imaging a target, while truncated Cauchy distribution function is employed to improve the quality of search result.

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