Study on Developing Regional Grid-Based Geoid Model Using GPS and Leveling Data

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Abstract: The height difference between the geodetic height h and the orthometric height H is called undulation N. The key issue of transforming the GPS height is to determine the undulation value accurately. If the undulation N for a point determined by a GPS receiver can be estimated in the field, then the 3-D geocentric coordinate in WGS-84 can be transformed to local coordinate system and orthometric height in real-time.

In this paper an algorithm to develop a regional grid-based geoid model using GPS data (e.g. ellipsoidal height) and the spiritual leveling data (e.g. orthometric height) is proposed. In brief, the proposed algorithm includes the following steps: (1) establishing the functional relationship between the point's plane coordinates (such as Northing and Easting) and the undulation (N) using back-propagation artificial neural network according to the measured GPS data and leveling data; (2) developing a regional grid-based geoid (undulation) model using the simulated grid plane coordinates with constant grid interval (e.g. 1500 m) and the generated functional relationship from step 1; (3) developing an undulation interpolation algorithm using the grid-based geoid model given a point's plane coordinates, and developing a computer program according this algorithm, then transforming this program to a pocket PC; (4) estimating the interested point's undulation in the field using the pocket PC and transforming the GPS height to orthometric height in real-time. Data from a series of 2,065 first-order bench marks (including GPS and leveling) around Taiwan are used to test the proposed algorithm. The test results show that the GPS height transformation accuracy using the grid-based geoid is on the order of 4 cm. The proposed algorithm and the detailed test results will be presented in this paper.
Keywords: GPS Height, Artificial Neural Network (ANN), Orthometric Height, Geometric Height, Undulation

1. Introduction

The coordinate system of GPS is the World Geodetic System of 1984 (WGS-84). Positions determined by GPS receivers are expressed in geocentric coordinate or geodetic coordinate defined by WGS-84 ellipsoid, but in engineering application, those coordinates need to be transformed to local coordinate system (such as Northing and Easting), and ellipsoidal (geodetic) heights (h) need to be transferred to physical heights, such as orthometric heights (H). The height difference between the geodetic height h and the orthometric height H is called undulation N. If the undulation N for a point determined by a GPS receiver is available, then the 3-D geocentric coordinate in WGS-84 can be transformed to local coordinate system and orthometric height. And the orthometric height H is the difference of its geodetic height h and the undulation N.

In this paper a transformation method from GPS geodetic height (h) to orthometric height (H) using back-propagation (BP) artificial neural network is proposed. This transformation method avoids puzzle for determination of geoid. Besides, this method establishes a mapping relation between geodetic height and orthometric height by means of neural network learning function. A 3-layer BP neural network, one input layer, one hidden layer, and one output layer, is adopted to transform GPS geodetic height h to orthometric height H.

In order to test the proposed method, a series of 283 bench marks, located at the central of Taiwan, are selected as the test data. The test region is about 116 km in easting and 103 km in northing. The distance between two consecutive bench marks is about 2 km. In this paper, the concept and methodology of the proposed algorism are briefly described and the test results are presented.

2. GPS Height Transformation

Positions determined by GPS receivers are expressed in geocentric coordinate (X, Y, Z) or geodetic coordinate $(\phi, \lambda, \mathrm{h})$ defined by WGS-84 ellipsoid. But in engineering application, those coordinates need to be transformed to local coordinate system (such as Northing and Easting), and ellipsoidal (geodetic) heights (h) need to be transferred to physical heights, such as orthometric heights (H). Note that the geodetic height h and the orthometric height H are referred to the surface of WGS-84 ellipsoid and the surface of the local geoid (mean sea level) respectively. Usually, these two surfaces are not coincide from point to point due to the fact that the physical earth is different from the mathematical earth. Hence, the height difference between the geodetic height h and the orthometric height H is called undulation N. The following equation establishes the relationship between h and H:

$$h = H + N \quad (1)$$

where h is the geodetic height; H is the orthometric height; and N is the undulation. All h, H, and N are in units of meters.

If the undulation N for a point determined by a GPS receiver is available, then the geodetic height h determined by GPS can be transformed to orthometric height H by the following equation:

$$H = h - N \quad (2)$$

From now on, the procedure of transforming the geodetic height h determined by GPS to orthometric height H is defined as GPS height transformation. Based on the foregoing discussion, the key issue of GPS height transformation is how to determine the corresponding undulation N of each point on the earth surface.

One of the methods of determining undulation N is called "curve fitting method". Supposed there are n reference points with known undulations in one area. Using a polynomial surface to fit these known undulations, then finding the coefficient terms of the polynomial by least square adjustment method. The following equation is an example of curve fitting method :

$$N(x,y) = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 \quad (3)$$

where x, y are the plane coordinates of a point ; N(x,y) is the corresponding undulation to point (x, y); $a_0 \cdots a_5$ are those polynomial coefficients.

For a new point p(x,y) determined by GPS, whose N can be determine by equation (3). Then, the orthometric height H can be computed by equation (2).

3. Artificial Neural Network

Artificial neural networks (ANNs) are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As in nature, the network function is determined largely by the connections between elements. We can train a neural network to perform a particular function by adjusting the values of the connections (weights) between elements. Commonly neural networks are adjusted, or trained, so that a particular input leads to a specific target output. The network is adjusted, based on a comparison of the output and the target, until the network output matches the target. Typically many such input/target pairs are used, in this supervised learning, to train a network (Demuth and Beale, 2002).

Neural networks have been trained to perform complex functions in various fields of application including pattern recognition, identification, classification, speech, vision and control systems. Back-propagation (BP) was created by generalizing the Widrow-Hoff learning rule to multiple-layer networks and nonlinear differentiable transfer functions. Input vectors and the corresponding target vectors are used to train a network until it can approximate a function, associate input vectors with specific output vectors, or classify input vectors in an appropriate way as defined by you. Networks with biases, a sigmoid layer, and a linear output layer are capable of approximating any function with a finite number of discontinuities (ibiz.).

4. GPS Height Transformation Using ANN

4.1 Basic Concept

If a point coordinates are determined by GPS, then its geodetic height h can be computed accordingly. Besides, its orthometric height H is determined by spiritual leveling method. Then, the corresponding undulation N can be computed by the following equation:

$$N = h - H \quad (4)$$

These points with geodetic height h and orthometric height H available are defined as reference points. Supposed there are n reference points in an interested region, we can use the reference points set $P = \{P_1, P_2, \cdots, P_n\}$ to train the BP artificial neural network (ANN).

$$P_i = (x_i, y_i, N_i) \quad i = 1,2,\cdots,n \tag{5}$$

where (x, y) are the plane coordinates of reference point I; N is the corresponding undulation N to point (x, y); i indicates the reference point number.

After trained by the reference point set $P = \{P_1, P_2, \cdots, P_n\}$, the BP ANN establishes the relationship between the input layer (x_i, y_i) and the output layer (N_i):

$$N = Geoid(x, y) \tag{6}$$

Once the ANN has been trained, then, simulate the network response to new inputs (x_i, y_i) using equation (6). Finally, compute the orthometric height H using equation (2).

4.2 MATLAB ANN Toolbox

MATLAB® is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others. There are generally four steps in the training process using BP ANN: (1) Assemble the training data, (2) Create the network object, (3) Train the network, (4) Simulate the network response to new inputs (Demuth and Beale, 2002).

5. Developing Regional Grid-Based Geoid Model Using GPS and Levelling Data

In order to develop a regional grid-based geoid model using GPS data (e.g. ellipsoidal height) and the spiritual leveling data (e.g. orthometric height), an algorithm is proposed. In brief, the proposed algorithm includes the following steps: (1) establishing the functional relationship between the point's plane coordinates (such as Northing and Easting) and the undulation (N) using back-propagation artificial neural network according to the measured GPS data and leveling data; (2) developing a regional grid-based geoid (undulation) model using the simulated grid plane coordinates with constant grid interval (e.g. 1500 m) and the generated functional relationship from step 1; (3) developing an undulation interpolation algorithm using the grid-based geoid model given a point's plane coordinates, and developing a computer program according this algorithm, then transforming this program to a pocket PC; (4) estimating the interested point's undulation in the field using the pocket PC and transforming the GPS height to orthometric height in real-time.

According to the proposed algorithm, the grid-based geoid model of Taiwan area was developed as follows:

(1) Collecting the required point's information, that is, points with GPS heights and orthometric heights, around the Taiwan area. These information will be used to train the artificial neural networks (ANNs).
(2) Defining a simulated grid with constant width, e.g. 1000 m. Then, estimating the undulation value N for each node of the simulated grid with artificial neural networks (ANNs). After this step, the grid-based geoid model was created.
(3) Once the grid-based geoid model was created, if the coordinate of a point was known, then the undulation of this specific point can be computed using the following equations (Lin, 1998):

$$N^p\left(N_p, E_p\right) = \sum_{i=1}^{4} W_i(x_p, y_p) \times N^i \tag{7}$$

Where $\left(N_p, E_p\right)$ are the plane coordinates of the specific point; N^p is the estimated undulation of point with coordinates $\left(N_p, E_p\right)$; i=1, 2, 3, 4 represent the four nodes of a grid which includes the point with coordinates $\left(N_p, E_p\right)$, the right-upper node is numbered 1, then the nodes are numbered in counter-clockwise sequence; N^i is the undulation value of node i ; $\left(N_1, E_1\right)$ is the coordinates of node 3, which are the minimum coordinate values of this specific grid; $\left(N_2, E_2\right)$ is the coordinates of node 1, which are the maximum coordinate values of this specific grid; W is the weighing function (Junkins et al., 1973).

$$W(x,y) = x^2 y^2 (9 - 6x - 6y + 4xy) \tag{8}$$

$$W_1(x,y) = W(x,y) \tag{9}$$

$$W_2(x,y) = W(1-x,y) \tag{10}$$

$$W_3(x,y) = W(1-x,1-y) \tag{11}$$

$$W_4(x,y) = W(x,1-y) \tag{12}$$

$$\Delta E_p = E_p - E_1 \tag{13}$$

$$\Delta N_p = N_p - N_1 \tag{14}$$

$$x_p = \frac{\Delta E_p}{E_2 - E_1} \tag{15}$$

$$y_p = \frac{\Delta N_p}{N_2 - N_1} \tag{16}$$

(4) Developing a pocket PC software suite according to the procedures of steps 2 and 3 using some program developing package, such as Microsoft Visual Studio Net 2003.

6. Test Results and Discussions

6.1 Test Data

In order to establish a precise vertical control system based on new Taiwan vertical datum 2001 (TWVD2001), a series of 2,065 first-order bench marks are measured and computed around Taiwan during year 1999 to 2002. In addition to leveling measurement, the GPS surveying and gravity surveying are carried out on those bench marks. Each bench mark has its 3-D geocentric coordinate, orthometric heights (H), and gravity measurements. Therefore, each bench mark has two different height values: geodetic height h and orthometric height H. There 2 test data sets are used in this paper. (1) A series of 283 bench marks from TWVD2001, located at the central of Taiwan, are selected as the test data. The test region is about 116 km in easting and 103 km in northing. The distance between two consecutive bench marks is about 2 km. In order to test the performance of the BP artificial neural network, these 283 bench marks are divided into two groups, one group with 142 points are treated as reference points to train the neural network, and the other group with 141 points are treated as check points. (2) All 2,065 bench marks from TWVD2001 are selected as test data set 2. They are divided into two groups, one group with 1033 points are treated as reference points to train the neural network, and the other group with 1032 points are treated as check points.

6.2 Test Methods

A 3-layer BP neural network, one input layer, one hidden layer, and one output layer, is adopted to transform GPS geodetic height h to orthometric height H. The input vector consists of Easting and Northing values (x_i, y_i) of each reference point (or each check point in case of simulation).

The output vector consists of each reference point's undulation (N_i).The transfer functions for the hidden layer and the output layer are 'tansig' (hyperbolic tangent sigmoid transfer function) and 'purelin' (linear transfer function) respectively. The various train methods, such as Bayesian regulation back propagation, Levenberg-Marquadardt back propagation, etc., are tested and the numbers of neurons of the hidden layer are changed.

The reference points set was used to train the neural network, and the check points set was used to evaluate the estimated accuracy of GPS height transformation. The proposed method is also compared with other approaches, such as (1) fitting method that simulates undulation surface using quadratic polynomial, and (2) geoid interpolation model.

6.3 Test Results and Discussions

6.3.1 Test Results of Varied Training Functions Using Test Data Set 1

In the following tests, the numbers of neurons of the hidden layer are fixed to 15, and the train algorithms are changed accordingly. The test results are shown in Table 1. The 'training algorithm' indicates the adopted backpropagation training algorithm. There are four different training algorithms are tested, i.e. 'trainbr' (Bayesian regularization backpropagation), 'trainlm' (Levenberg-Marquardt backpropagation), 'traincgf' (Conjugate gradient backpropagation with Fletcher-Reeves updates), and 'traingdx' (Gradient descent with momentum and adaptive learning rate backpropagation) (Demuth and Beale, 2002). The term of 'ΔN' is defined by the following equation:

$$\Delta N_i = N_i^{known} - N_i^{simulate}, \quad i = 1,2,\cdots,n \tag{17}$$

where N_i^{known} is the known undulation N of check point i; $N_i^{simulate}$ is the simulated undulation N using the trained ANN; ΔN_i is the undulation difference between the known undulation and the simulated undulation. The above-mentioned parameters are in units of meters. '$\sigma_{\Delta N}$' indicates the standard deviation of all ΔN_i, in units of meters. 'iteration number' indicates the epochs taken when the ANN training performance met.

From Table 1, it is found that the iteration number of using 'trainbr' algorithm is minimum with 0.0439 m ($\sigma_{\Delta N}$). On the other hand, it is shown that the $\sigma_{\Delta N}$ of using 'trainlm' algorithm is minimum, however, with the maximum iteration number (20000). Therefore, it is recommended that the 'trainbr' algorithm be used to transform GPS height if both ANN training speed and accuracy are considered.

Table 1. Test results of varied training algorithms.

Training Algorithm	Iteration Number (epochs)	$\sigma_{\Delta N}$ (m)	Maximum ΔN (m)	Minimum ΔN (m)
trainbr	247	0.0439	0.1238	-0.2130
trainlm	20000	0.0342	0.0683	-0.1269
traincgf	733	0.0615	0.1248	-0.4279
traingdx	20000	0.0619	0.1362	-0.1938

6.3.2 Test Results of Varied Number of Neurons Using Test Data Set 1

In the following tests, the numbers of neurons of the hidden layer are changed from 5 to 50, while the training algorithm of 'trainbr' was used always. The test results are summarized in Table 2. From Table 2, it was found that the value of $\sigma_{\Delta N}$ decreases from 0.0510 m to 0.0331 m when the number of neurons increases from 5 to 40.

6.3.3 Accuracy Comparisons with Other Undulation Estimation Methods Using Test Data Set 1

In order to evaluate the performance of ANN on GPS height transformation, the same check points set was tested using other approaches, such as (1) fitting method that simulates undulation surface using quadratic polynomial, and (2) the undulation interpolation model of MOI (Ministry of Interior, Taiwan). The test results are summarized in Table3. 'BP ANN' means ANN with 'trainbr' training algorithm and 15 neurons was used to transform GPS height. 'Curve Fitting' indicates that the polynomial of equation (3) was used to fit the undulations of the reference points. 'MOI Model' shows that the MOI undulation interpolation model was used to estimate the undulations of those check points. From Table 3, it can be seen that the $\sigma_{\Delta N}$ value of 'BP ANN' is much smaller than those of other two estimation methods.

6.3.4 Interpolation Accuracy Evaluation with Grid-Based Geoid Model Using Test Data Set 2

In order to evaluate the interpolation accuracy with the grid-based geoid model of the Taiwan area, the test data set 2 was used. The grid-based geoid model was generated based on the algorithm described in section 5 using the 1033 reference points. Then the rest 1032 points are used to evaluate the interpolation accuracy of the grid-based geoid model. Two grid-based geoid models with different grid intervals, they are 1000 M and 1500 M respectively, was generated. Besides, each model was generated using different number of neurons and the accuracy was evaluated. The test results are shown in Table 4 and Table 5. From Table 4 and Table 5, it is found that: (1) the more of the number of neurons the better of the interpolation accuracy; (2) when the number of the neurons is larger than 30, then the interpolation accuracy comes to stable, says about 0.049 M; (3) the shorter the grid interval, the better the interpolation accuracy.

Table 2. Test results of varied number of neurons with 'trainbr' training algorithms.

Number of Neurons	Iteration Number (epochs)	$\sigma_{\Delta N}$ (m)	Maximum ΔN (m)	Minimum ΔN (m)
5	290	0.0510	0.1190	-0.1679
10	856	0.0412	0.1297	-0.1373
15	247	0.0439	0.1238	-0.2130
20	1252	0.0432	0.1205	-0.1693
25	658	0.0442	0.1239	-0.2212
30	786	0.0360	0.1294	-0.1039
35	798	0.0360	0.1294	-0.1039
40	1504	0.0331	0.0884	-0.0844
45	832	0.0437	0.1058	-0.1357
50	1404	0.0385	0.0967	-0.1340

Table 3. Test results of accuracy comparisons with other undulation estimation methods.

Estimation Method	$\sigma_{\Delta N}$ (m)	Maximum ΔN (m)	Minimum ΔN (m)
BP ANN	0.0439 M	0.1238	-0.2130
Curve Fitting	0.1988 M	0.8286	-0.6033
MOI Model	0.2007 M	0.4710	-0.3470

Table 4. Test results of varied number of neurons with grid interval 1000 M.

Number of Neurons	$\sigma_{\Delta N}$ (m)	Maximum ΔN (m)	Minimum ΔN (m)
5	0.1500	0.7038	-0.7476
10	0.0768	0.6724	-0.2951
20	0.0526	0.6623	-0.2325
30	0.0493	0.6201	-0.2133
40	0.0470	0.6196	-0.2186
50	0.0467	0.6214	-0.2504

Table 5. Test results of varied number of neurons with grid interval 1500 M.

Number of Neurons	$\sigma_{\Delta N}$ (m)	Maximum ΔN (m)	Minimum ΔN (m)
5	0.2566	0.9950	-0.7329
10	0.1164	0.7874	-1.0090
20	0.0568	0.6370	-0.5440
30	0.0658	0.6197	-0.9929
40	0.0573	0.6358	-0.7919
50	0.05681	0.6196	-1.5500

7. Conclusions

In order to transform the GPS height into orthometric height in real-time, a grid-based geoid model of Taiwan area was proposed using GPS and leveling data. A data set of 2065 bench marks from TWVD2001, Taiwan was used to test the proposed algorithm.

The preliminary test results indicate that: (1) the 'trainbr' algorithm should be used to transform GPS height if both ANN training speed and accuracy are considered, (2) the value of $\sigma_{\Delta N}$ decreases from 0.0510 m to 0.0331 m when the number of neurons increases from 5 to 40, if the 'trainbr' training algorithm was adopted, (3) the performance of GPS height transformation using BP artificial neural network is better than the other two estimation methods. The $\sigma_{\Delta N}$ values for 'BP ANN', 'Curve Fitting', and 'MOI Model' are 0.044 m, 0.199 m, and 0.201 m respectively, (4) the GPS height transformation accuracy using the grid-based geoid is on the order of 4 cm.

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